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RESEARCH AND DEVELOPMENT TECHNICAL REPORT
CORADCOM—78-5

MODELING AND ANALYSIS OF MAN-MACHINE
INTERFACE INFORMATION.

Final Technical Rept.,

Richard J. D'Accardi
CENTER FOR COMMUNICATIONS SYSTEMS

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Considering the premise that communications systems are composed of electronic equipment, environmental effects, and the human element, one becomes aware of the difficulty in assuming knowledge of every aspect of a system. In modeling or analyzing a system, one should characterize each facet of the system separately by defining the most influential factors as stochastic variables and then proceed to estimate parameters and structure statistical models of operational efficiency. One example of a non-stationary physical process that influences communications systems is derived from the response of a

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systems operator to various levels of environmental stress. Due to the random nature of this phenomenon, two logical approaches to forecasting and interpreting experimental results with respect to operator performance seems to lie within the realms of non-linear modeling and non-stationary time-series modeling.

The main objectives of this report are as follows:

(i) The design of a statistical experiment is presented for man-machine interface studies aimed at the standard teletypewriter terminal and the optical display terminal. The structuring of this experiment includes time dependent and time independent formulations.

(ii) Various classical and non-parametric statistical analyses are presented which provide insight as to what degree the environmental variables of ambient light and acoustic noise affect operator response.

(iii) Two statistical models have been proposed for the analysis and interpretation of man-machine interface data. First, a time-independent structuring is presented for predicting errors as a function of environmental variables, and an optimal scheme is given for the determination of the errors. Secondly, a time-series approach to man-machine interface problems is introduced and difference equations have been formulated for the purpose of forecasting errors as a function of time.

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1. MAN-MACHINE INTERFACE ANALYSIS

1.1 INTRODUCTION

Information gained in evaluating and solving man-machine interface problems that occur in complex communications systems is extremely important to systems engineers committed to the mission of the design and fabrication of future generations of equipment. Sophisticated systems of command and control, computer-aided man-in-the-loop systems (e.g., manned spacecraft), human response to audio and visual displays, pattern recognition, man-computer languages, and many other facets, are of concern where an operator must perform a control task, or decision task. At present there is some ongoing work oriented towards man-machine interfaces which span the projected needs of the armed forces, [1], [2], [3], [4], [5], [6]. Work in progress by many agencies generally deals with evaluation of complex system interfaces, assessment of operator performance capabilities for a wide variety of tasks (including performance as a function of ambient light alone), analysis of manual functions into tasks, analysis of human control functions, and the physical and psychological characteristics which affect the assessment of operator performance capabilities. Much of the ongoing work concerns the psychological and physiological aspects of command and control, weapons systems, logistics, and communications.

Examination of ongoing research in these areas indicate that there is no clear-cut procedure to evaluate the human subsystem in sophisticated communications systems or the effects of environmental stress on operator performance. Army communications requirements in a tactical situation often require 24-hour operations. Personnel are required to work either on standard, or unpatterned, and frequently extended duty schedules in a variety of environments, each characterized by multiple stresses occurring in a random manner. In an effort to seek novel measures of man-machine interfaces which occur in communications systems and to enhance the design of future families of equipment, this section will address teletype operator perform-ance as the environmental factors of *ambient light* and *acoustic noise* are varied. The discussion will center around an experiment conducted at Fort Monmouth, New Jersey, during April and May, 1975. The experiment was designed to answer the following questions:

i) Is there a significant deterioration of operator performance (committed errors) as the environmental factors of *ambient light* and *acoustic noise* are varied?

ii) Does acoustic noise affect operator performance more than ambient light in a deteriorating environment?

iii) Is operator performance significantly different for the optical display terminal as opposed to the teletypewriter terminal?

iv) Given certain ambient light and acoustic noise levels, can one predict the number of transcription errors that will be committed for a given terminal?

v) Given the operating conditions, can one specify the levels of light and sound so as to minimize the number of committed errors?

It is an aim of this study to answer the above questions with respect to contingency table analysis, three-factor analysis of variance, and multiple non-linear regression.

In the next section, 1.2, the design of the experiment is described which includes the manner in which the information was obtained, the types of equipment employed, and the environment under which the experiment was conducted. The levels of the variables considered are typical of those encountered in a tactical environment.

There are two ways in which one can proceed to answer the questions with respect to the dependence of the environmental variables and the number of committed errors (operator performance). Contingency table analysis was employed in the one case, which is covered in detail in section 1.3; a comparison was made of the second case to the classical three-factor analysis of variance techniques as described in section 1.4.

An extensive search was conducted to find the best non-linear regression model that best characterizes the relationship between the dependent and independent variables. The best model was judged by the criterion of minimum residual variance. Section 1.5 describes the regression models tested. A summary of results, conclusions, and recommendations for future work is presented in section 1.6.

1.2 DESIGN OF THE EXPERIMENT

The significance of acoustic noise and ambient light on operator performance was investigated using both an optical display transmission device (see

figure 1.1), and a standard teletypewriter (see figure 1.2). The visual display terminal is designed to interface with computers or store-and-forward devices. Primarily, it is a developmental equipment intended to visually present messages on a CRT display where an operator can see and correct his message prior to transmission.

The experiment consisted of testing the transcription accuracy of six experienced communications-center operators under 16 different combinations of environment. Ambient light was varied at four levels, ranging from 24 foot-candles to 3 foot-candles; and acoustic noise was concurrently varied at four sound pressure levels ranging from 55 dBa to 95 dBa. Sound pressure level (SPL), measured in dBa, is in reference to .0002 dynes/cm^2 and is considered the threshhold of hearing. This reference is roughly equivalent to a leaf "falling" on a quiet day. The 55 dBa level was considered the quiet condition where only the inherent noise from the terminal equipment, sound room noise, and thermal noise were recorded. The 95 dBa level represented an extremely annoying and distracting "pink" noise. The noise-power per unit frequency for this type of noise is inversely proportioned to frequency over a specified range and slopes down at 3 dB per octave from 20 Hz to 20 KHz. These characteristics are more common to conference-type noise where the higher and lower frequency components characterize motor and equipment noises. Pink noise was also used because it has relatively constant energy per octave-bandwidth. The chosen ambient light levels of 24, 12, 6, and 3 foot-candles, respectively, represented successively deteriorating room lighting conditions. Throughout the testing, the brightness of the optical display was constant.

For each test the operator was required to type his name, treatment combination, and date as part of the message (see figure 1.3). The messages for the experiment consisted of forty random-letter word groups of five characters each. They were derived through a random number generator and an alphanumeric conversion. No message was a duplicate, nor were they duplicated by any of the operators on either terminal equipment. The random letter format was used so that the operator could not identify or recognize routine words and, therefore, would have to concentrate to avoid making transcription errors. The aim of the experiment was to vary the environmental variables and to observe the transcription accuracy of each operator utilizing the visual display terminal as a function of time. The response variable,

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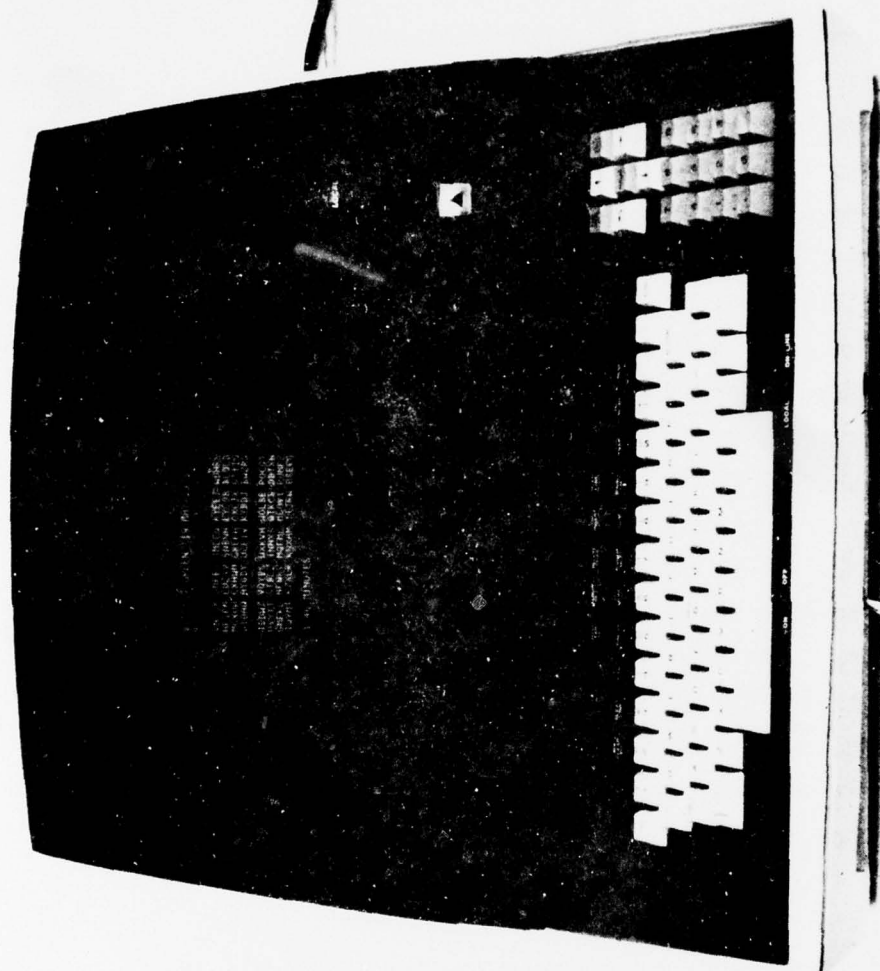
FIGURE 1.1 OPTICAL DISPLAY TERMINAL
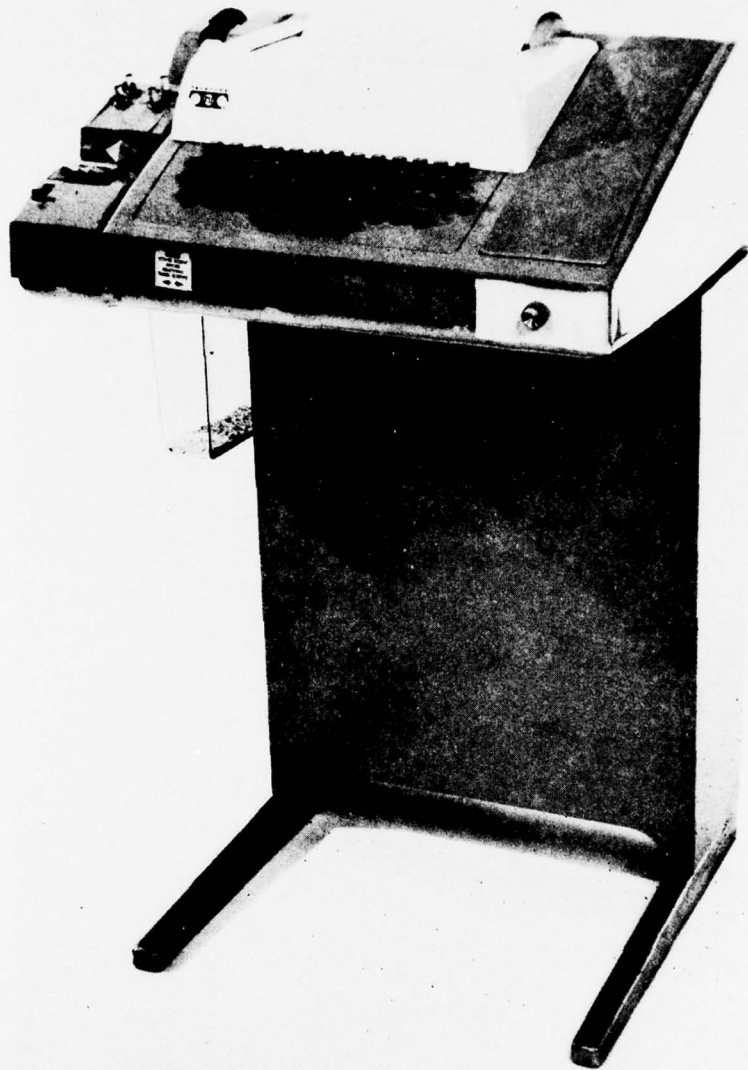
FIGURE 1.2 TELETYPEWRITER TERMINAL

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BETTY J GREEN 19 MAY 1975
4.2

VHTGX GMFBO JMLCO KYEET WDHRS
HEIPN EUZXV SNBNH EYCOB BLQLD
MCXUW CXHWM CREFH OXCJX GTIIQ
GIOAQ AYDGY UCEIV KYBDJ BWZRG

JBDWZ VQYBS HXKVH WOLBE POCGT
UYYTT STBLZ INNXS QTKCB DDTXN
GMEKL NFNKV MQFML MEOVI PUMPZ
TQYMT KZXLR KNXAH ITJRW ESERG

1 46 MINUTES

FIGURE 1.3 TYPICAL MESSAGE FORMAT

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accuracy (number of committed errors), was the measure of transcription errors that each operator committed per four-second interval. The errors considered were the following:

1) transposition
2) missing letter
3) extra letter
4) incorrect space
5) extra line feed
6) missing word groups
7) wrong letter
8) line out of sequence (skipped line inserted after detection)
9) word group out of sequence

The results were compared to an acceptable operator norm, i.e., typing a message format on a standard teletype terminal under the same conditions. Each operator was tested alone in four sessions, each session having been programmed for eight random environmental combinations, four for each terminal equipment (see table 1.1). The tests were alternated between the optical display unit and the standard teletypewriter. This was done to reduce the effects of learning. A thirty-minute familiarization period was given each operator prior to the tests, and a standard instruction sheet was distributed during this period to insure uniform orientation with the equipment and with the purpose and procedure of the experiment.

1.3 CONTINGENCY TABLE ANALYSIS [7], [8]

The following analyses will attempt to answer whether or not:

i) acoustic noise affects committed errors more than ambient light level,

ii) there is a difference in committed errors when the optical display terminal is used as opposed to the teletypewriter,

iii) operator performance deteriorates significantly as the light and sound levels are varied.

Specifically, the purpose of this analysis is to show how the variation of light and sound levels affects the number of transcription errors made by communications operators using two different tactical communications terminals.

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TABLE 1.1

TREATMENT SCHEDULE PER OPERATOR

Session	Run	Environmental Treatment * Combinations	
		Optical Display Terminal	Teletype Terminal
I	1	1,4	3,1
	2	4,3	4,4
	3	3,2	2,2
	4	2,1	1,3
II	5	3,1	4,1
	6	4,4	1,2
	7	2,2	3,4
	8	1,3	2,3
III	9	4,1	2,4
	10	1,2	3,3
	11	3,4	1,1
	12	2,3	4,2
IV	13	2,4	1,4
	14	3,3	4,3
	15	1,1	3,2
	16	4,2	2,1

* Treatment = (Ambient Light Level, Acoustic Noise Level)

Ambient Light		Acoustic Noise	
Level	Value	Level	Value
1	24 ft-candles	1	55 dBa
2	12 ft-candles	2	70 dBa
3	6 ft-candles	3	80 dBa
4	3 ft-candles	4	95 dBa

A trainee was chosen as a member of the operator group to provide insight into the number of committed errors he would commit versus those of the more experienced personnel. There was indeed a significant difference in his performance, as will be shown later in this section. As a result, the data generated by the trainee was deleted from the final analysis. Heretofore, the trainee will be referred to as the "fifth" replication. The analysis, therefore, was broken down into the following major categories:

i) section 1.3.1 - contingency table analysis, all subjects, for the teletypewriter terminal.

ii) section 1.3.2 - contingency table analysis, all subjects, for the optical display terminal.

iii) section 1.3.3 - contingency table analysis with the fifth replication deleted for the teletypewriter terminal.

iv) section 1.3.4 - contingency table analysis with the fifth replication deleted for the optical display terminal.

1.3.1 Contingency Table Analysis, All Subjects, For The Teletypewriter Terminal

The data was formed into a two-way contingency table by summing the number of errors for each replication in each of the sixteen cells (environmental combinations). For ease of description, the ambient light level will be referred to as factor A, and the acoustic noise level will be referred to as factor B. Thus, for the teletypewriter terminal, the following hypotheses were tested:

$$H_0: \text{ factor A is independent of factor B,}$$

versus

$$H_1: \text{ factor A is dependent on factor B,}$$

with the test statistic:

$$\chi^2 = \sum_{j=1} \sum_{i=1} \frac{(n_{ij} - \frac{n_{i \cdot} n_{\cdot j}}{n})^2}{\frac{n_{i \cdot} n_{\cdot j}}{n}} \sim \chi^2 (a-1)(b-1) ,$$

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where a = number of levels of factor A,

    b = number of levels of factor B,

n_{ij} = number of observations in the ij^{th} cell,

$n_{i.}$ = sum of observations in the ith row,

$n_{.j}$ = sum of observations in the jth column, and

    n = total number of observations.

The calculated test statistic resulted in χ^2 = 17.41, and the appropriate critical values or points at various levels of significance with $(a-1)(b-1) = (3)(3) = 9$ degrees of freedom are:

$$\alpha = .05, \quad \chi^2_{(9)} = 16.9$$
$$\alpha = .025, \quad \chi^2_{(9)} = 19.0$$
$$\alpha = .01, \quad \chi^2_{(9)} = 21.7$$

Therefore, one rejects the null hypothesis at the α = .05 level.

To determine the level or levels of either factor for which the number of errors is minimum, the row and column totals of either the entire table or appropriate partitions should be considered. Considering factor A alone, from the following hypotheses:

$$H_0: \text{ row sums are equal, i.e., } P_{1.} = P_{2.} = P_{3.} = P_{4.}$$

versus

$$H_1: \text{ row sums are unequal, i.e., } P_{1.} \neq P_{2.} \neq P_{3.} \neq P_{4.}$$

where $P_{i.}$ is the probability of an error in the ith row; and using the test statistic:

$$\chi^2 = \sum_{i=1}^{a} \frac{(n_{i.} - n/a)^2}{n/a} \sim \chi^2_{(a-1)},$$

the calculated value was found to be χ^2 = 10.43. In this case, the critical points at various levels of significance, α, with a-1 = 3 degrees of freedom are:

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$$\alpha = .05, \quad \chi^2_{(3)} = 7.81$$
$$\alpha = .025, \quad \chi^2_{(3)} = 9.35$$
$$\alpha = .01, \quad \chi^2_{(3)} = 11.3$$

This indicates quite strongly that there is a difference in the levels of factor A with respect to the number of errors. This further indicates that there exists a level, or group of levels, that results in a minimum number of committed errors. Through a similar process, the row totals were compared, one at a time, to all the others to determine at which level the most significant difference occurred. The test and the test statistic were similar to the above with a=2 being the number tested in this case. When comparing the smallest row total to the largest, the calculated $\chi^2 = 10.08$ with a-1 degree of freedom. A strong, significant difference was observed in this case. When considering the smallest row total and the second largest, $\chi^2 = 5.22$ with 1 degree of freedom, which indicated a significant difference at the $\alpha = .025$ level. Finally, when considering the smallest row total and the second smallest, $\chi^2 = 2.83$ with 1 degree of freedom, which strongly indicated no significant difference in the number of errors committed. Thus, one can conclude that at 3 foot-candles and 6 foot-candles of illumination, the number of transcribed errors are minimum.

Now, considering factor B alone, the following hypotheses were formulated:

H_0: column sums are equal, i.e., $P_{.1} = P_{.2} = P_{.3} = P_{.4}$

versus

H_1: column sums are not equal, i.e., $P_{.1} \neq P_{.2} \neq P_{.3} \neq P_{.4}$,

where $P_{.j}$ is the probability of an error in the jth column. Using the test statistic:

$$\chi^2 = \sum_{j=1}^{b} \frac{(n_{.j} - \frac{n}{b})^2}{\frac{n}{b}} \sim \chi^2(b-1) ,$$

yielded a calculated value of $\chi^2 = 13.99$ with $(b-1) = 3$ degrees of freedom, which strongly indicated a significant difference in the levels of factor B

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at the α = .05 level with respect to the number of errors. Again, this indicates that there is a level, or group of levels, at which the number of committed errors is minimum. It was further observed that only the level with the least number of errors compared to that with the most errors showed any significant difference, or acceptance of H_0. Therefore, the conclusion for the standard teletypewriter terminal is that the factors of ambient light and acoustic noise are dependent, and that the minimum number of committed errors occur when the ambient light is at or above 6 foot-candles, and, concurrently, when the noise level is at or below 80 dBa.

1.3.2 Contingency Table Analysis, All Subjects, For The Optical Display Terminal

For the optical display terminal, the procedure described in section 1.3.1 was again duplicated with the hypotheses formulated as:

$$H_0: \text{ factor A is independent of factor B}$$

versus

$$H_1: \text{ factor A is dependent on factor B.}$$

The calculated test statistic for this case was χ^2 = 9.1 with 9 degrees of freedom, which strongly indicated the independence of the two factors (acceptance of the null hypotheis at the α = .05 level). Continuing on, for factor A alone, i.e., the significance of the levels of A on operator performance, the appropriate hypotheses are:

$$H_0: \text{ row sums are equal; } P_1. = P_2. = P_3. = P_4.$$

versus

$$H_1: \text{ row sums are unequal; } P_1. \neq P_2. \neq P_3. \neq P_4.$$

The calculated test statistic was χ^2 = 3.17 with 3 degrees of freedom, which indicated no significant difference in the levels of factor A at χ = .05. This shows that varying the levels of factor A (ambient light) does not signficantly affect the number of committed errors.

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For factor B alone, the following hypotheses were tested:

H_0: Column sums are equal, i.e., $P_{.1} = P_{.2} = P_{.3} = P_{.4}$

versus

H_1: Comumn sums are unequal, i.e., $P_{.1} \neq P_{.2} \neq P_{.3} \neq P_{.4}$

The calculated value of the test statistic obtained was $\chi^2 = 14.29$ with 3 degrees of freedom. This, of course, indicated a strong difference in the levels of factor B at $\alpha = .05$ with respect to the number of committed errors, and further indicated a level, or group of levels, at which a minimum (or maximum) number of errors occurred.

To detect which of the levels is most significant, the following tests were made:

H_0: Least column sum equal to the largest column sum, $P_{.1} = P_{.4}$

versus

H_1: Least column sum unequal to the largest column sum, $P_{.1} \neq P_{.4}$

In this case, the calculated test statistic $\chi^2 = 10.24$ with 1 degree of freedom indicated a significant difference between these sound levels, i.e., 55 dBa and 95 dBa for $\alpha = .05$. Repeating this procedure again for:

H_0: Least column sum equal to second largest column sum, $P_{.1} = P_{.3}$

versus

H_1: Least column sum unequal to second largest column sum, $P_{.1} \neq P_{.3}$

The test statistic, $\chi^2 = 1.41$ with 1 degree of freedom, indicated no significant difference in the number of committed errors at the $\alpha = .05$ level. This, of course, implies no difference in the effects of these levels, i.e., 55 dBa, 70 dBa, and 80 dBa. The conclusion of this series of tests was that the factors were independent, with the minimum number of errors occurring when the level of acoustic noise was *at* or *below* 80 dBa. The level of light did not, however, affect the number of errors produced. This was apparently due to the fact that the human eye is highly adaptable to deteriorating light conditions, at least to the 3 foot-candle level.

1.3.3 Contingency Table Analysis With 5th Replication Deleted For The Teletypewriter Terminal

What remained, then, was to show the effect of the trainee (fifth replication) on the results. Thus, the above series of tests were again duplicated, with the fifth replication deleted, as follows:

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H_0:   factor A independent of factor B

versus

H_1:   factor A not independent of factor B,

for which the test statistic $\chi^2 = 6.42$ with 9 degrees of freedom indicated acceptance of H_0 at the $\alpha = .05$ level; that is, the two factors are independent. Testing again with respect to the levels of ambient light:

H_0:   there is no difference in row totals, $P_1. = P_2. = P_3. = P_4.$

versus

H_1:   there is a difference between row totals, $P_1. \neq P_2. \neq P_3. \neq P_4.$,

the test statistic $\chi^2 = 9.4$ with 3 degrees of freedom indicated a rejection of H_0 at a level of significance, $\alpha = .05$. Thus, there is a range of levels at which the number of errors is minimum. For the specfic levels of A:

H_0:   there is no difference between the smallest and largest row totals, $P_1. = P_4.$

versus

H_1:   there is a difference between the smallest and largest row totals, $P_1. \neq P_4.$

The test statistic $\chi^2 = 6.49$ with 1 degree of freedom indicated a significant difference at $\alpha = .05$.

Comparing the smallest and second largest row totals, $P_1.$ and $P_3.$, $\chi^2 = 1.382$ with 1 degree of freedom, indicated no difference at $\alpha = .05$, i.e., $P_1. = P_3.$. Thus, the range of levels at which the committed errors are minimum is 6-24 foot-candles.

To investigate the error behavior with respect to acoustic noise, we have:

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H_0:   there is no difference between column totals, $P_{.1} = P_{.2} = P_{.3} = P_{.4}$

versus

H_1:   there is a difference between column totals, $P_{.1} \neq P_{.2} \neq P_{.3} \neq P_{.4}$

In this case, the test statistic, $\chi^2 = 31.29$ with 3 degrees of freedom, indicates that there is a significant difference at $\alpha = .05$ between the levels of factor B (column sums) with respect to the number of committed errors. This again affirms that there exists a range of levels at which the number of errors is minimal. Testing for these specific levels, we again compare the smallest column total, $P_{.1}$, to the largest, $P_{.4}$. Here, $\chi^2 = 23.35$ with 1 degree of freedom, indicates there is a significant difference at $\alpha = .05$. Testing again for the smallest column total and the second largest, $P_{.3}$, $\chi^2 = 20.15$ with 1 degree of freedom. This indicates, as before, that $P_{.1} \neq P_{.3}$ at $\alpha = .05$. Finally, testing for the difference between the smallest column total, $P_{.1}$, and the second smallest, $P_{.2}$, $\chi^2 = 3.34$ with 1 degree of freedom. Here, H_0 is accepted at the $\alpha = .05$ level. Thus, there is no significant difference between them; that is $P_{.1} = P_{.2}$. This means that the range of level B at which the number of errors is minimum occurs between 55 dBa and 70 dBa.

1.3.4 Contingency Table Analysis With 5th Replication Deleted For The Optical Display Terminal

A similar procedural approach was implemented to study the optical display terminal. One begins by testing the hypothesis:

H_0:   factor A is independent of factor B

versus

H_1:   factor A is dependent on factor B

the test statistic $\chi^2 = 14.84$, with 9 degrees of freedom, indicates that factor A is independent of factor B (accept H_0) at $\alpha = .05$. Next, considering the levels of factor A alone, we test:

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H_0: there is no difference in row totals.

versus

H_1: there is a difference in row totals.

The calculated test statistic $\chi^2 = 2.96$ with 3 degrees of freedom, indicates no significant difference in the levels of factor A at the $\alpha = .05$ level. This means that varying the level of ambient light did not cause the number of committed errors to vary significantly as one would expect. The test to define the range of factor A levels is, therefore, not necessary since the previous test indicated acceptance of the null hypothesis.

Now, testing the hypotheses with respect to the levels of factor B alone, we have:

H_0: there is no difference between column totals

versus

H_1: there is a difference between column totals

In this case, $\chi^2 = 12.78$ with 3 degrees of freedom indicates a difference between the levels with respect to the number of transcribed errors at the $\alpha = .05$ level. This, of course, indicates that there is a level, or group of levels, at which the number of errors is minimal. To define these levels (or range), the following hypotheses were tested:

H_0: there is no difference between the smallest and largest column totals, $P_{.1} = P_{.4}$

versus

H_1: there is a difference between the smallest and largest column totals, $P_{.1} \neq P_{.4}$

Here, $\chi^2 = 13.25$ with 1 degree of freedom indicates there is a significant difference at $\alpha = .05$. The comparison of the smallest and second largest column totals, $P_{.1}$ and $P_{.3}$, where $\chi^2 = 2.62$ with 1 degree of freedom, indicates acceptance of H_0 at the $\alpha = .05$ level. This further defines the range of levels at which the minimum number of errors are committed, namely, 55 dBa to 80 dBa.

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Thus, from this series of tests we conclude that the factors of ambient light and acoustic noise are independent, with the minimum number of errors occurring when the sound level is below 80 dBa. The level of light, however, does not significantly affect operator performance. The human eye is highly adaptable to deteriorating light conditions, at least to the 3 foot-candle level.

The conclusion evident from sections 1.3.3 and 1.3.4, in comparing the results with those of sections 1.3.1 and 1.3.2 is that, due to the discrepancies in the results, the trainee (operator with only three months experience) should not be considered in the analysis of the performance of experienced operators. Therefore, the conclusions of sections 1.3.3 and 1.3.4 should be considered the most valid.

1.4   ANALYSIS OF VARIANCE

In view of the fact that the data was limited in the number of observations, the contingency table analysis was implemented to answer the relevant questions without having to place a probability structure on the observed information. Secondly, although the assumptions of normality cannot be justified with the limited amount of information available, an analysis of variance, [9], [10], was performed on the man/machine interface data so as to obtain comparable results on the questions posed in section 1.1.

1.4.1 Two-Factor Analysis of Variance

Considering the teletypewriter terminal first, the initial model:

$$Y = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$$

was taken, where:

Y = the dependent variable which represents the number of transcribed errors,

μ = the overall mean effect,

α_i = the effect of the levels of factor A (ambient light) for the ith level,

β_j = the effect of the levels of factor B (acoustic noise) for the jth level,

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γ_{ij} = the effect of the interaction of A and B,

ε_{ijk} = the experimental error; that is, the extent to which the observed data and the general model disagree for k replications.

From this formulation, and from the teletypewriter terminal data, the following ANOVA table was obtained:

	df.	Sum of Square, S.S.	M.S. - Mean Square = $\frac{S.S.}{d.f.}$
factor A	a-1 = 3	55.9375	18.6458
factor B	b-1 = 3	75.0375	25.0125
Interaction AB	(a-1)(b-1) = 9	107.5125	11.9458
error	ab(k-1) = 64	1312	20.5
Total	abk-1 = 79	1550.4875	

First, testing for the significance of the interaction AB, the following hypotheses were formulated:

$$H_0: \quad \gamma_{ij} = 0 \text{ against } H_1: \quad \gamma_{ij} \neq 0.$$

The appropriate test statistic yields:

$$\frac{MSAB}{MSE} = .5827 \text{ and } F(x)_{(a-1)(b-1), ab(k-1)} = F(.05) = 2.04.$$
$$ 9,64$$

Thus, the null hypothesis that the interaction of A and B is not significant at $\alpha = .05$ is accepted. This shows that the model is additive as opposed to the possibility of being multiplicative. One can, therefore, combine the γ_{ij}, and ε_{ijk}. This results in a model of the form:

$$Y = \mu + \alpha_i + \beta_j + \varepsilon'_{ijk} ,$$

where $\varepsilon'_{ijk} = \gamma_{ij} + \varepsilon_{ijk}$. The new sum squared error term for this model is:

$$SSE' = SSAB + SSE = 1419.5125 ,$$

with (a-1)(b-1) + ab(k-1) = 73 degrees of freedom.

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Now, testing for the significance of the levels of factor B, namely β_j,

$$H_0: \beta_j = 0 \text{ versus } H_1: 3_j \neq 0,$$

we have $\frac{MSB}{MSE}$, $= 1.286$ and $F_{3,73}(\alpha) \approx 2.74$. Thus, we accept the null hypothsis at the $\alpha = .05$ level of significance. This result shows that varying the levels of factor B will not significantly affect the number of committed errors. Now, since β_j is not significant, one may combine the β_j and ε'_{ijk}. This results in a new model:

$$Y = \mu + \alpha_i + \varepsilon''_{ijk}$$

where

$$\varepsilon''_{ijk} = \varepsilon'_{ijk} + \beta_j = \beta_j + \gamma_{ij} + \varepsilon_{ijk}$$

and the SSE'' = SSB + SSAB + SSE = 1494.55 with $(a-1)(b-1) + b-1 + ab(k-1)$ = 76 degrees of freedom.

Testing for the significance of the levels of factor A; that is

$$H_0: \alpha_i = 0 \text{ versus } H_1: \alpha_i \neq 0,$$

we have $\frac{MSA}{MSE}$, , $= .948$ and $F_{3,76}(\alpha) \approx 2.74$, which implies that H_0 must be accepted at the $\alpha = .05$ level of significance. This shows that varying the levels of factor A will not significantly affect the number of errors produced. The apparent conclusion of these tests is that the model which characterizes the differences in the number of errors in each cell is:

$$Y = \mu + \varepsilon'''_{ijk}$$

where

$$\varepsilon'''_{ijk} = \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}.$$

Repeating the above procedure for the *optical display* terminal with the same initial model, hypotheses, and parameters as previously defined, the initial ANOVA table is:

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	Degrees of Freedom, d.f.	Sums of Squares, s.s.	Mean Squares, m.s.
Factor A	3	19.6375	6.5458
Factor B	3	88.4375	29.479
Interaction AB	9	58.1125	6.457
Error	64	1098.	17.156
Total	79	1264.1875	

Testing for the significance of the interaction, we have:

$$H_0: \; \gamma_{ij} = 0 \text{ versus } H_1: \; \gamma_{ij} \neq 0$$

The calculated test statistic in this case is: $\dfrac{MSAB}{MSE} = 0.376$ and

$F_{9,64}(\alpha) = 2.04$. Thus, we accept H_0 at the $\alpha = .05$ level of significance. This again indicates that the model is additive, and the interaction term, γ_{ij}, can be combined with ε_{ijk}. Now, the revised model is written as:

$$Y = u + \alpha_i + \beta_j + \varepsilon'_{ijk}$$

where $\varepsilon'_{ijk} = \gamma_{ij} + \varepsilon_{ijk}$, and the resulting $SSE' = 1419.5125$ with 73 degrees of freedom.

Next, we consider the significance of acoustic noise; that is,

$$H_0: \; \beta_j = 0 \text{ versus } H_1: \; \beta_j \neq 0$$

The calculated test statistic is $\dfrac{MSB}{MSE'} = 1.516$ and $F_{3,73}(\alpha) = 2.74$. Therefore, we accept H_0 at $\alpha = .05$. The indication is that varying the levels of factor B will not significantly affect the number of errors. Thus, combining β_j and ε'_{ijk} yields the revised model:

$$Y = u + \alpha_i + \varepsilon''_{ijk}$$

where $\varepsilon''_{ijk} = \beta_j + \gamma_{ij} + \varepsilon_{ijk}$, and the $SSE'' = 1244.55$ with 76 degrees of freedom.

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Finally, we test the levels of factor A, the α_i; that is,

$$H_0: \quad \alpha_i = 0 \text{ versus } H_1: \quad \alpha_i \neq 0$$

one obtains the test statistic: $\frac{MSA}{MSE}_{\langle\langle} = 0.3997$ and $F(\alpha)_{3,76} \simeq 2.74$. Thus, we
accept H_0 at the $\alpha = .05$ level of significance. Here again, as with the tele-
typewriter, it is indicated that varying the levels of factor A will not
significantly affect the number of errors. The conclusion reached from these
tests is that the model which describes the difference in the sums of errors
in each cell for both the optical display terminal and the teletypewriter
terminal is:

$$Y = \mu + \varepsilon'''_{ijk}$$

This means that there is no apparent effect on operator performance for either
terminal as the levels of ambient light and acoustic noise are varied within
the chosen ranges. It should be noted that the criteria for analysis of
variance (that the observations in each cell are normally distributed and
that the variances in each cell are homogeneous), were not substantiated.
Therefore, the results of sections 1.3.3 and 1.3.4 should prevail.

In view of these results, the next step in the overall analysis was to
identify and to sort out any additional factors that may have affected the
experimental error. The following sub-section addresses the incorporation of
the most logical third effect, namely, performance differences between
operators.

1.4.2 Three-Factor Analysis of Variance

Since there was an appreciable difference in the number of committed
errors among the subjects tested, an additional variable was defined, namely,
operator difference. Thus, for this analysis a three-factor analysis of vari-
ance, [11]* was performed with factors A, B, and C, representing light, sound,
and operator difference, respectively. The respective levels of each main
factor a = 4 (the levels of ambient light), b = 4 (the levels of acoustic
noise), and c = 5 (the number of operators, excluding the trainee).

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In a completely randomized experimental design, the model for the three-factor analysis is given by [11]:

$$Y_{ijkl} = u + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkl}$$

(1.1)

$i = 1, 2, \ldots a; \ j = 1, 2, \ldots b; \ k = 1, 2, \ldots c;$ and $l = 1, 2, \ldots n$, where Y_{ijkl} is the dependent variable representing the number of errors, u is the overall mean effect, α_i, β_j, and γ_k are the main effects; $(\alpha\beta)_{ij}$, $(\alpha\gamma)_{ik}$, and $(\beta\gamma)_{jl}$ are the two-factor interaction effects that have the same interpretation as in a two-factor experiment. The term $(\alpha\beta\gamma)_{ijk}$ is called the three-factor interaction effect, namely, a term that represents a nonadditivity of the $(\alpha\beta)_{ij}$ over the different levels of the factor C. The sum of all main effects is zero and the sum over any subscript of the two- and three-factor interaction effects is zero. In many situations, these higher-order interactions are insignificant and their mean squares reflect only random variation.

The general philosophy of the analysis is the same as that for a one- or two-factor experiment. The sum of squares is partitioned into eight terms, each representing a source of variation from which one obtains independent estimates of the common variance, σ^2, when all the main effects and interaction effects are zero. If the effects of any given factor or interaction are not all zero, then the mean square will estimate the error variance plus a component due to the effect in question.

The computational procedure for obtaining the sums of squares in a three-factor analysis of variance requires the following notation:

$T\ldots$ = sum of all $abcn$ observations

$T_{i\ldots}$ = sum of the observations for the ith level of factor A

$T_{.j..}$ = sum of the observations for the jth level of factor B

$T_{..k.}$ = sum of the observations for the kth level of factor C

$T_{ij..}$ = sum of the observations for the ith level of A and the jth level of B

$T_{i.k.}$ = sum of the observations for the ith level of A and the kth level of C

$T_{.jk.}$ = sum of the observations for the jth level of B and k^{th} level of C

$T_{ijk.}$ = sum of the observations for the $(ijk)^{th}$ treatment combination.

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This notation is used in constructing the following two-way tables of totals and subtotals:

A	B 1	B 2	...	B b	Total
1	$T_{11k\cdot}$	$T_{12k\cdot}$	...	$T_{1bk\cdot}$	$T_{1\cdot k\cdot}$
2	$T_{21k\cdot}$	$T_{22k\cdot}$	...	$T_{2bk\cdot}$	$T_{2\cdot k\cdot}$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
a	$T_{a1k\cdot}$	$T_{a2k\cdot}$	...	$T_{abk\cdot}$	$T_{a\cdot k\cdot}$
Total	$T_{\cdot 1k\cdot}$	$T_{\cdot 2k\cdot}$	...	$T_{\cdot bk\cdot}$	$T_{\cdot\cdot k\cdot}$

$k = 1, 2 \ldots, c$

A	B 1	B 2	...	B b	Total
1	$T_{11\cdot\cdot}$	$T_{12\cdot\cdot}$	...	$T_{1b\cdot\cdot}$	$T_{1\cdot\cdot\cdot}$
2	$T_{21\cdot\cdot}$	$T_{22\cdot\cdot}$	...	$T_{2b\cdot\cdot}$	$T_{2\cdot\cdot\cdot}$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
a	$T_{a1\cdot\cdot}$	$T_{a2\cdot\cdot}$	...	$T_{ab\cdot\cdot}$	$T_{a\cdot\cdot\cdot}$
Total	$T_{\cdot 1\cdot\cdot}$	$T_{\cdot 2\cdot\cdot}$	...	$T_{\cdot b\cdot\cdot}$	$T_{\cdot\cdot\cdot\cdot}$

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A	C				Total
	1	2	...	c	
1	$T_{1\cdot1\cdot}$	$T_{1\cdot2\cdot}$	$\cdots$	$T_{1\cdot c\cdot}$	$T_{1\cdots}$
2	$T_{2\cdot1\cdot}$	$T_{2\cdot2\cdot}$	$\cdots$	$T_{2\cdot c\cdot}$	$T_{2\cdots}$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
a	$T_{a\cdot1\cdot}$	$T_{a\cdot2\cdot}$	$\cdots$	$T_{a\cdot c\cdot}$	$T_{a\cdots}$
Total	$T_{\cdots1}$	$T_{\cdots2}$	$\cdots$	$T_{\cdots c}$	$T_{\cdots\cdot}$

B	C				Total
	1	2	...	c	
1	$T_{\cdot11\cdot}$	$T_{\cdot12\cdot}$	$\cdots$	$T_{\cdot1c\cdot}$	$T_{\cdot1\cdots}$
2	$T_{\cdot21\cdot}$	$T_{\cdot22\cdot}$	$\cdots$	$T_{\cdot2c\cdot}$	$T_{\cdot2\cdot\cdot}$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
b	$T_{\cdot b1\cdot}$	$T_{\cdot b2\cdot}$	$\cdots$	$T_{\cdot bc\cdot}$	$T_{\cdot b\cdot\cdot}$
Total	$T_{\cdots1}$	$T_{\cdots2}$	$\cdots$	$T_{\cdots c}$	$T_{\cdots\cdot}$

The sum of squares are computed by using the following computational formulas, [11]:

$$SST = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \sum_{l=1}^{n} y_{ijkl}^2 - \frac{T_{\cdots}^2}{abcn}$$

$$SSA = \frac{\sum_{i=1}^{a} T_{i\cdots}^2}{bcn} - \frac{T_{\cdots}^2}{abcn}$$

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$$\text{SSB} = \frac{\sum\limits_{j=1}^{b} T^2_{\cdot j \cdot \cdot}}{acn} - \frac{T^2_{\cdot \cdot \cdot \cdot}}{abcn}$$

$$\text{SSC} = \frac{\sum\limits_{k=1}^{c} T^2_{\cdot \cdot k \cdot}}{abn} - \frac{T^2_{\cdot \cdot \cdot \cdot}}{abcn}$$

$$\text{SS(AB)} = \frac{\sum\limits_{i=1}^{a} \sum\limits_{j=1}^{b} T^2_{ij \cdot \cdot}}{cn} - \frac{\sum\limits_{i=1}^{a} T^2_{i \cdot \cdot \cdot}}{bcn} - \frac{\sum\limits_{j=1}^{b} T^2_{\cdot j \cdot \cdot}}{acn} + \frac{T^2_{\cdot \cdot \cdot \cdot}}{abcn}$$

$$\text{SS(AC)} = \frac{\sum\limits_{i=1}^{a} \sum\limits_{k=1}^{c} T^2_{i \cdot k \cdot}}{bn} - \frac{\sum\limits_{i=1}^{a} T^2_{i \cdot \cdot \cdot}}{bcn} - \frac{\sum\limits_{k=1}^{c} T^2_{\cdot \cdot k \cdot}}{abn} + \frac{T^2_{\cdot \cdot \cdot \cdot}}{abcn}$$

$$\text{SS(BC)} = \frac{\sum\limits_{j=1}^{b} \sum\limits_{k=1}^{c} T^2_{\cdot jk \cdot}}{an} - \frac{\sum\limits_{j=1}^{b} T^2_{\cdot j \cdot \cdot}}{acn} - \frac{\sum\limits_{k=1}^{c} T^2_{\cdot \cdot k \cdot}}{abn} + \frac{T^2_{\cdot \cdot \cdot \cdot}}{abcn}$$

$$\text{SS(ABC)} = \frac{\sum\limits_{i=1}^{a} \sum\limits_{j=1}^{b} \sum\limits_{k=1}^{c} T^2_{ijk \cdot}}{n} - \frac{\sum\limits_{i=1}^{a} \sum\limits_{j=1}^{b} T^2_{ij \cdot \cdot}}{cn} - \frac{\sum\limits_{i=1}^{a} \sum\limits_{k=1}^{c} T^2_{i \cdot k \cdot}}{bn}$$

$$- \frac{\sum\limits_{j=1}^{b} \sum\limits_{k=1}^{c} T^2_{\cdot jk \cdot}}{an} + \frac{\sum\limits_{i=1}^{a} T^2_{i \cdot \cdot \cdot}}{bcn} + \frac{\sum\limits_{j=1}^{b} T^2_{\cdot j \cdot \cdot}}{acn} + \frac{\sum\limits_{k=1}^{c} T^2_{\cdot \cdot k \cdot}}{abn}$$

$$- \frac{T^2_{\cdot \cdot \cdot \cdot}}{abcn} \quad ,$$

and the SSE, as usual, is obtained by subtraction. The computations in an analysis-of-variance problem for a three-factor experiment with n replications are summarized in the following table, [11]:

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TABLE 1.2

THREE-WAY ANOVA

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Computed f
Main Effects				
A	SSA	$a-1$	s_1^2	$f_1 = \dfrac{s_1^2}{s^2}$
B	SSB	$b-1$	s_2^2	$f_2 = \dfrac{s_2^2}{s^2}$
C	SSC	$c-1$	s_3^2	$f_3 = \dfrac{s_3^2}{s^2}$
Two-factor Interactions				
AB	SS(AB)	$(a-1)(b-1)$	s_4^2	$f_4 = \dfrac{s_4^2}{s^2}$
AC	SS(AC)	$(a-1)(c-1)$	s_5^2	$f_5 = \dfrac{s_5^2}{s^2}$
BC	SS(BC)	$(b-1)(c-1)$	s_6^2	$f_6 = \dfrac{s_6^2}{s^2}$
Three-factor Interactions				
ABC	SS(ABC)	$(a-1)(b-1)(c-1)$	s_7^2	$f_7 = \dfrac{s_7^2}{s^2}$
Error	SSE	$abc(n-1)$	s^2	
Total	SST	$abcn-1$		

For a three-factor analysis with a single replicate, one sets n=1 and uses the ABC interaction for the error sum of squares. In this case, one assumes that the ABC interaction is zero and that SS(ABC) represents variation due only to experimental error and is, therefore, the estimate of the error variance.

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Using this methodology, the following ANOVA table was obtained for the teletypewriter terminal data:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Computed f
A	55.938	3	18.646	1.912
B	75.037	3	25.012	2.564
C	813.674	4	203.418	20.854
AB	107.512	9	11.946	1.225
AC	125.020	12	10.418	1.068
BC	22.126	12	1.844	0.189
ABC (error)	351.161	36	9.754	-----

The associated critical values of the F distribution for the $\alpha = .05$ level of significance are:

$$F_{3,36}(\alpha) = 2.88$$

$$F_{4,36}(\alpha) = 2.65$$

$$F_{9,36}(\alpha) = 2.16$$

$$F_{12,36}(\alpha) = 2.04$$

In this case, n=1 replication per cell, the trainee was deleted, and the error term, as previously mentioned, is the mean square of the ABC interaction term. One can conclude from the computed f statistics that the operator differences are indeed significant, and that light, sound, and the second order interactions are insignificant.

This shows that the model is additive with respect to factors A, B, C, and the interactions, as opposed to the possibility of being multiplicative. Since we are primarily interested in the main factors A, B, and C, one can combine the $(\alpha\beta)_{ij}$, $(\alpha\gamma)_{ib}$, $(\beta\gamma)_{jk}$, and $(\alpha\beta\gamma)_{ijk}$. This results in a model of the form:

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$$y = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon'_{ijk} ,$$

where

$$\varepsilon'_{ijk} = (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk}$$

The new ANOVA table and new sum squared error term are shown below:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Computed f
A	75.037	3	25.012	2.849
B	55.938	3	18.646	2.124
C	813.674	4	203.418	23.168
Error (AB + BC + AC + ABC)	605.819	69	8.780	----

The associated critical values of the F distribution for the $x = .05$ level of significance are:

$$F_{3,69}(\alpha) = 2.76 \text{ and } F_{4,69}(\alpha) = 2.53$$

Thus, one can conclude that varying the levels of ambient light (factor A), does not significantly affect the number of errors committed, while varying the levels of acoustic noise does have a significant effect on operator performance. Therefore, sorting out the operator differences (which were significant as expected) achieved a decision comparable to the contingency table analysis of section 1.3. Now, for the second case, considering the optical display unit data, the following ANOVA table applies (with the F distribution critical points the same as for the teletypewriter data):

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
A	19.637	3	6.546	0.531
B	88.438	3	29.479	2.392
C	487.998	4	121.999	9.899
AB	58.110	9	6.457	0.524
AC	49.799	12	4.150	0.337
BC	116.497	12	9.708	0.788
ABC(error)	443.679	36	12.324	-----

The conclusions drawn from this information are similar to those of the first case. With the interaction effects not significant and thereby additive, the following model was formulated: $y = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon'_{ijk}$, for $n=1$ observations per cell, where $\varepsilon'_{ijk} = (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk}$ as in the first case. The following ANOVA table was constructed for the new model:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
A	19.637	3	6.546	0.676
B	88.438	3	29.479	3.045
C	487.998	4	121.999	12.600
Error (AB+AC+BC+ABC)	668.080	69	9.682	------

For the $\alpha = .05$ level of significance, both operator differences and acoustic noise had a significant effect on operator performance. Therefore, sorting out the operator differences as a third variable (which was again significant as expected) achieved decisions comparable to the contingency table analysis, namely, that acoustic noise significantly affects the number of errors committed, while ambient light (within the 3 to 24 foot-candle range) does not. In view of the results obtained through this section, the next step in

the analysis is to develop a scheme to predict or to precisely characterize operator performance as a function of light and noise. To this end, the following section will address a non-linear regression technique specifically geared to this type of problem.

1.5 A NON-LINEAR REGRESSION MODEL FOR MAN-MACHINE INTERFACE

In this section, an acceptable model [12], [13], to predict operator performance is presented with the appropriate technique to determine the environmental combination of ambient light and acoustic noise that generally causes a minimum number of committed errors. Various linear, multiple linear, and non-linear models were tested for both the optical display terminal and the teletypewriter terminal. What follows is considered to be the best model for both experiments. The criterion used for choosing the best model was the minimum SSE (sum of squares error) where

$$SSE = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2,$$

and Y_i = observed errors

$\hat{Y}_i$ = predicted errors

1.5.1 Non-Linear Modeling:

The general model that best describes the observed data is of the form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_1^2 + \beta_5 X_2^2$$

$$+ \beta_6 X_1 X_2^2 + \beta_7 X_1^2 X_2 + \beta_8 X_1^2 X_2^2 + \beta_9 X_1^3 + \beta_{10} X_2^3 + \varepsilon_j$$

$$(1.2)$$

where

Y = average number of errors (operator performance) per cell

X_1 = ambient light level,

X_2 = acoustic noise level,

β_i = model coefficients, i = 0, 1, ... 10,

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ε_j = experimental error, $j = 1, \ldots, n$ (the extent to which the observed data and the model disagree, where ε_j's are independent and $\varepsilon \sim N(0, \sigma^2 I)$), and

$n = 16$.

The estimated values of the coefficients, error variance, correlation, and appropriate F statistic for both terminals are summarized in table 1.3.

TABLE 1.3 ESTIMATED PARAMETERS FOR NON-LINEAR MODELS

Parameter	Optical Display Terminal	Teletypewriter Terminal
β_0	34.7500	-7.793
β_1	.5092	-6.365
β_2	-1.0840	1.018
β_3	- .0399	.1588
β_4	.0359	.1668
β_5	.0137	- .02055
β_6	.0002373	- .0007769
β_7	.001990	- .004906
β_8	- .000011	.00002257
β_9	.003293	.001425
β_{10}	.000053	.0001133
SSE	5.136	3.389
S_ε^2	1.027	.6779
$F_{(MODEL)}$	2.735	6.536
$R_{y\hat{y}}^2$	.8455	.9289
$R_{y\hat{y}}$	.9195	.9638

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In the case of the optical display terminal, the F statistic indicates a possible overabundance of variables. In the case of the teletypewriter terminal, the small SSE, large R^2_{yy}, and relatively small F statistic, indicate an acceptable model.

Now, we begin to investigate the possibility of reducing the above model consistent with the principle of parsimony. That is, we eliminate the variables in the model that do not significantly contribute to the dependent variable. The procedure used to form the reduced models was the "forward selection procedure," [9]* which begins with the variable, X_i, that has the highest correlation $\rho x_i y$. Next, the partial correlation coefficients of the remaining x_j and y, $\rho(x_j y \mid x_i)$, $j \neq i$, are calculated. The x_j with the greatest $\rho(x_j y \mid x_i)$ is selected to enter the regression equation. This process is continued, and as each variable is entered into the equation, the multiple correlation coefficient R^2_{yy} and the partial F test value for the most recent entry are examined. In the first case, one checks to assure a relatively insignificant change in R^2_{yy}, and, secondly, whether or not the inserted variable has taken up a significant amount of variation over the previous variables in the regression model. When the partial F test becomes insignificant (the SSE is sufficiently reduced) and R^2_{yy} is not very different from the "full model," the process is terminated. The reduced model, therefore, is that which contains all significant variables plus the first two insignificant variables to accommodate any error due to the estimates.

Based on the general model previously stated (see equation 1.2), the appropriate reduced models that characterize operator performance for both terminals are as follows:

      i) for the optical display terminal:

$$y = \beta'_0 + \beta'_1 X_1 + \beta'_2 X_1 X_2^2 + \beta'_3 X_1^3 + \beta'_4 X_2^3 + \varepsilon$$

where: $\beta'_0 = 10.63$,

$\beta'_1 = -0.1239$,

$\beta'_2 = 0.000028$,

$\beta'_3 = -0.0002202$,

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$$\beta'_4 = 0.000008367,$$

$$\text{with SSE} = 8.678,$$

$$S_\varepsilon^2 = 0.7389,$$

$$F_{(MODEL)} = 7.783,$$

$$R_{y\hat{y}}^2 = 0.7389,$$

$$R_{y\hat{y}} = 0.8596$$

ii) for the teletypewriter terminal:

$$Y = \beta'_0 + \beta'_1 X_1 + \beta'_2 X_1 X_2 + \beta'_3 X_1^2 X_2 + \beta'_4 X_1^3 + \beta'_5 X_2^3 + \varepsilon$$

where: $\beta'_0 = 3.211,$

$$\beta'_1 = -1.365,$$

$$\beta'_2 = 0.03532,$$

$$\beta'_3 = -0.001288,$$

$$\beta'_4 = 0.002123,$$

$$\beta'_5 = -0.000004273,$$

with SSE = 7.63,

$$S_\varepsilon^2 = 0.763$$

$$F_{(MODEL)} = 10.5$$

$$R_{y\hat{y}}^2 = 0.34$$

$$R_{y\hat{y}} = 0.9165$$

The reduced models now provide the capability to predict the number of transcribed errors, given the desired combination of ambient light and acoustic noise. Further, they also permit a reasonable extrapolation outside the tested environmental limits to simulate additional data.

1.5.2 Optimal Light and Sound Levels

One can now attempt to find the light-sound combination that causes the least number of errors to be committed. One method of accomplishing this task is the standard method of differential calculus. This entails taking the derivatives of the prediction equation with respect to X_1 and X_2, respectively, setting them equal to zero, and solving the resultant system of equations simultaneously for X_1 and X_2 with the proper constraints. The method used instead is to simply evaluate the predicted value of Y for ordered pairs, (X_1, X_2), where X_1 assumes all integer values from 1 to 26, and X_2 assumes even integer values from 50 to 100. These ranges of X_1 and X_2 were chosen based upon the levels of X_1 and X_2 used in the experiment.

The predicted Y values, i.e., the predicted number of errors, were calculated for the environemtnal combinations described in section 1.2 for the optical display data (using the reduced model) to obtain the matrix of table 1.4. Visual examination of this matrix shows that the minimum number of errors, i.e., 4.4, will occur at a light level of 24 foot-candles and a concurrent noise level of 54 dBa, or, if we are willing to extrapolate slightly outside the region from which data has been obtained, the absolute minimum, 3.8, occurs at 26 foot-candles and 50 dBa. Thus, one can conclude that the minimum number of errors committed on the optical terminal (in the region for which data was taken) occurs at the minimum sound and maximum light combination, that is, 24 foot-candles/55 dBa.

A similar matrix of predicted errors was computed for the reduced tele-typewriter model, and is shown in table 1.5. In this case, visual examination shows that the minimum number of predicted errors occurs at a light level of about 16-17 foot-candles and at a concurrent sound level of about 55 dBa. In both cases (optical display and teletypewriter) the results of the minima were expected. It is to be noted, however, that in a tactical situation, the environmental factors of ambient light and acoustic noise are far from optimal. The true worth of the matrices (predictions) is to show for a wide variety of the environmental factors, X_1 and X_2, what one can expect, that is, how well experienced communicators will perform.

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Table 1.4

PREDICTED ERROR PERFORMANCE FOR VARIOUS LEVELS OF AMBIENT LIGHT AND ACOUSTIC NOISE FOR THE OPTICAL DISPLAY TERMINAL.

ACOUSTIC NOISE LEVEL -dBa

	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100
26	3.8	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.5	4.7	4.9	5.1	5.3	5.6	5.8	6.1	6.4	6.8	7.2	7.5	7.9	8.4	8.8	9.3	9.8	10.4
25	4.1	4.1	4.1	4.2	4.3	4.4	4.5	4.6	4.8	4.9	5.1	5.3	5.5	5.8	6.1	6.3	6.7	7.0	7.3	7.7	8.1	8.5	9.0	9.5	10.0	10.5
24	4.4	4.4	4.4	4.5	4.6	4.6	4.7	4.9	5.0	5.2	5.3	5.5	5.8	6.0	6.2	6.5	6.8	7.2	7.5	7.9	8.3	8.7	9.1	9.6	10.1	10.6
23	4.6	4.7	4.7	4.7	4.8	4.9	5.0	5.1	5.2	5.4	5.5	5.7	5.9	6.2	6.4	6.7	7.0	7.3	7.6	8.0	8.4	8.8	9.2	9.6	10.1	10.6
22	4.9	4.9	4.9	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.7	5.9	6.1	6.3	6.5	6.8	7.1	7.4	7.7	8.1	8.4	8.8	9.3	9.7	10.2	10.6
21	5.1	5.1	5.1	5.1	5.2	5.3	5.3	5.4	5.6	5.7	5.8	6.0	6.2	6.4	6.7	6.9	7.2	7.5	7.8	8.1	8.5	8.9	9.3	9.7	10.2	10.6
20	5.3	5.3	5.3	5.3	5.4	5.4	5.5	5.6	5.7	5.8	6.0	6.1	6.3	6.5	6.7	7.0	7.3	7.6	7.8	8.2	8.5	8.9	9.3	9.7	10.2	10.6
19	5.4	5.4	5.4	5.4	5.5	5.5	5.6	5.7	5.8	5.9	6.1	6.2	6.4	6.6	6.8	7.0	7.3	7.6	7.9	8.2	8.5	8.9	9.3	9.7	10.1	10.6
18	5.6	5.6	5.6	5.6	5.6	5.6	5.7	5.8	5.9	6.0	6.1	6.3	6.4	6.6	6.8	7.1	7.3	7.6	7.9	8.2	8.5	8.9	9.2	9.6	10.1	10.5
17	5.7	5.7	5.7	5.7	5.7	5.6	5.7	5.9	5.9	6.1	6.2	6.3	6.5	6.7	6.9	7.1	7.3	7.6	7.8	8.1	8.4	8.7	9.1	9.5	9.9	10.4
16	5.8	5.8	5.8	5.8	5.8	5.8	5.8	5.9	6.0	6.1	6.2	6.3	6.5	6.6	6.8	7.0	7.3	7.5	7.8	8.1	8.4	8.7	9.1	9.5	9.9	10.3
15	5.9	5.9	5.9	5.8	5.8	5.8	5.9	5.9	6.0	6.1	6.2	6.3	6.5	6.6	6.8	7.0	7.2	7.4	7.7	8.0	8.3	8.6	9.0	9.4	9.7	10.2
14	5.9	5.9	5.9	5.9	5.9	5.9	5.9	6.0	6.0	6.1	6.2	6.3	6.5	6.6	6.8	7.0	7.2	7.4	7.7	7.9	8.2	8.5	8.9	9.2	9.6	10.0
13	6.0	5.9	5.9	5.9	5.9	5.9	5.9	5.9	6.0	6.1	6.2	6.3	6.4	6.6	6.7	6.9	7.1	7.3	7.6	7.8	8.1	8.4	8.7	9.1	9.4	9.8
12	6.0	5.9	5.9	5.9	5.9	5.9	5.9	5.9	6.0	6.1	6.1	6.2	6.4	6.5	6.6	6.8	7.0	7.2	7.5	7.7	8.0	8.3	8.6	8.9	9.3	9.7
11	6.0	5.9	5.9	5.9	5.8	5.8	5.8	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.6	6.7	6.9	7.1	7.3	7.6	7.8	8.1	8.4	8.7	9.1	9.5
10	6.0	5.9	5.9	5.8	5.8	5.8	5.8	5.8	5.9	5.9	6.0	6.1	6.2	6.3	6.5	6.6	6.8	7.0	7.2	7.4	7.7	7.9	8.2	8.5	8.9	9.2
9	5.9	5.9	5.9	5.8	5.8	5.7	5.7	5.8	5.8	5.9	5.9	6.0	6.1	6.2	6.3	6.5	6.6	6.8	7.0	7.3	7.5	7.8	8.0	8.3	8.7	9.0
8	5.9	5.8	5.8	5.7	5.7	5.7	5.7	5.8	5.7	5.8	5.8	5.9	6.0	6.1	6.2	6.3	6.5	6.7	6.9	7.1	7.3	7.6	7.8	8.1	8.4	8.8
7	5.9	5.8	5.8	5.7	5.7	5.6	5.6	5.6	5.7	5.7	5.7	5.8	5.9	6.0	6.1	6.2	6.4	6.5	6.7	6.9	7.1	7.4	7.6	7.9	8.2	8.5
6	5.9	5.7	5.7	5.6	5.6	5.6	5.6	5.5	5.6	5.6	5.6	5.7	5.7	5.8	5.9	6.1	6.2	6.3	6.5	6.7	6.9	7.2	7.4	7.7	8.0	8.3
5	5.8	5.7	5.6	5.6	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.7	6.9	7.2	7.4	7.7	8.0
4	5.8	5.7	5.6	5.5	5.4	5.4	5.4	5.3	5.3	5.4	5.4	5.4	5.5	5.5	5.6	5.7	5.8	6.0	6.1	6.3	6.5	6.7	6.9	7.2	7.5	7.7
3	5.7	5.6	5.5	5.4	5.4	5.3	5.3	5.2	5.2	5.2	5.2	5.3	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.5	6.7	6.9	7.2	7.5
2	5.6	5.5	5.4	5.3	5.3	5.2	5.2	5.1	5.1	5.1	5.1	5.2	5.2	5.2	5.3	5.4	5.5	5.6	5.7	5.9	6.1	6.2	6.5	6.7	6.9	7.2
1	5.6	5.4	5.3	5.3	5.2	5.1	5.1	5.0	5.0	5.0	5.0	5.0	5.0	5.1	5.1	5.2	5.3	5.4	5.5	5.7	5.8	6.0	6.2	6.4	6.7	6.9

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Table 1.5

PREDICTED ERROR PERFORMANCE FOR VARIOUS LEVELS OF AMBIENT LIGHT AND ACOUSTIC NOISE FOR THE TELETYPEWRITER TERMINAL.

ACOUSTIC NOISE LEVEL - dBa

AMBIENT LIGHT LEVEL	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100
26	6.9	6.9	6.9	7.0	7.0	7.0	7.0	7.0	7.0	6.9	6.9	6.9	6.8	6.8	6.7	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.8	5.7	5.5
25	5.6	5.7	5.8	5.9	5.9	6.0	6.1	6.1	6.2	6.2	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.2	6.2	6.2	6.1	6.0	6.0	5.9	5.8
24	4.6	4.7	4.8	5.0	5.1	5.2	5.3	5.5	5.6	5.7	5.7	5.8	5.9	6.0	6.0	6.1	6.1	6.2	6.2	6.2	6.2	6.2	6.2	6.2	6.1	6.1
23	3.7	3.9	4.0	4.2	4.4	4.6	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.3	6.4	6.4	6.4	6.5	6.5
22	2.9	3.2	3.4	3.6	3.9	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.4	5.6	5.7	5.9	6.0	6.2	6.2	6.4	6.5	6.5	6.7	6.8	6.8	6.9
21	2.4	2.6	2.9	3.2	3.4	3.7	4.0	4.2	4.4	4.7	4.9	5.1	5.3	5.5	5.7	5.9	6.1	6.3	6.4	6.6	6.7	6.9	7.0	7.1	7.2	7.3
20	1.9	2.2	2.5	2.9	3.2	3.4	3.7	4.0	4.3	4.6	4.8	5.1	5.3	5.5	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.2	7.3	7.5	7.6	7.7
19	1.6	2.0	2.3	2.6	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.6	5.9	6.1	6.4	6.6	6.8	7.1	7.3	7.5	7.7	7.8	8.0	8.2
18	1.4	1.8	2.1	2.5	2.9	3.2	3.5	3.9	4.2	4.5	4.8	5.2	5.5	5.7	6.0	6.3	6.6	6.8	7.1	7.3	7.6	7.8	8.0	8.2	8.4	8.6
17	1.3	1.7	2.1	2.5	2.8	3.2	3.6	3.9	4.3	4.6	4.9	5.3	5.6	5.9	6.2	6.5	6.8	7.1	7.3	7.6	7.9	8.1	8.3	8.6	8.8	9.0
16	1.3	1.7	2.1	2.5	2.9	3.3	3.6	4.0	4.4	4.7	5.1	5.4	5.8	6.1	6.4	6.7	7.0	7.3	7.6	7.9	8.1	8.4	8.6	8.9	9.1	9.3
15	1.4	1.8	2.2	2.6	3.0	3.4	3.8	4.1	4.5	4.9	5.2	5.6	5.9	6.3	6.6	6.9	7.2	7.5	7.8	8.1	8.4	8.7	8.9	9.2	9.4	9.6
14	1.5	1.9	2.3	2.7	3.1	3.5	3.9	4.3	4.7	5.0	5.4	5.8	6.1	6.4	6.8	7.1	7.4	7.7	8.0	8.3	8.6	8.9	9.1	9.4	9.6	9.9
13	1.7	2.1	2.5	2.9	3.3	3.7	4.1	4.5	4.8	5.2	5.6	5.9	6.3	6.6	6.9	7.3	7.6	7.9	8.2	8.5	8.8	9.0	9.3	9.5	9.8	10.0
12	1.9	2.3	2.7	3.1	3.5	3.9	4.3	4.6	5.0	5.4	5.7	6.1	6.4	6.7	7.1	7.4	7.7	8.0	8.3	8.6	8.8	9.1	9.4	9.6	9.8	10.1
11	2.1	2.5	2.9	3.3	3.7	4.1	4.4	4.8	5.2	5.5	5.8	6.2	6.5	6.8	7.1	7.4	7.7	8.0	8.3	8.6	8.8	9.1	9.3	9.6	9.8	10.0
10	2.4	2.8	3.1	3.5	3.9	4.2	4.6	4.9	5.3	5.6	5.9	6.2	6.6	6.9	7.2	7.4	7.7	8.0	8.3	8.6	8.8	9.0	9.2	9.6	9.8	10.0
9	2.6	3.0	3.3	3.7	4.0	4.4	4.7	5.0	5.3	5.6	5.9	6.3	6.5	6.8	7.1	7.4	7.6	7.9	8.1	8.4	8.6	8.8	9.0	9.2	9.4	9.6
8	2.9	3.2	3.5	3.8	4.2	4.5	4.8	5.1	5.4	5.6	5.9	6.2	6.5	6.7	7.0	7.2	7.4	7.7	7.9	8.1	8.3	8.5	8.6	8.8	9.0	9.1
7	3.1	3.4	3.7	3.9	4.2	4.5	4.7	5.0	5.3	5.6	5.8	6.0	6.3	6.5	6.7	6.9	7.1	7.3	7.5	7.7	7.8	8.0	8.1	8.3	8.4	8.5
6	3.2	3.5	3.7	4.0	4.2	4.5	4.7	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.5	6.7	6.9	7.0	7.1	7.3	7.4	7.5	7.6	7.7	7.8
5	3.3	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.1	5.3	5.5	5.6	5.8	5.9	6.0	6.1	6.3	6.4	6.4	6.5	6.6	6.7	6.7	6.8	6.8
4	3.4	3.6	3.7	3.9	4.1	4.2	4.4	4.5	4.6	4.7	4.9	5.0	5.1	5.2	5.3	5.4	5.4	5.5	5.5	5.6	5.6	5.7	5.7	5.7	5.7	5.7
3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.2	4.3	4.4	4.4	4.5	4.5	4.5	4.6	4.6	4.6	4.6	4.6	4.5	4.5	4.5	4.4	4.3
2	3.2	3.3	3.4	3.4	3.5	3.5	3.5	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.5	3.5	3.4	3.4	3.3	3.2	3.1	3.0	2.9	2.8
1	3.0	3.0	3.0	3.0	3.0	3.0	2.9	2.9	2.9	2.8	2.8	2.7	2.6	2.6	2.5	2.4	2.3	2.2	2.1	1.9	1.8	1.7	1.5	1.3	1.2	1.0

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1.6 SUMMARY OF RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

The table on the following page has been formulated to summarize the analysis of the preceding sections. The indications from the tests of independence are clear, that the effect of the trainee did in fact bias the initial results. The decision, therefore, to delete the trainee's results from the analysis was indeed valid. One can note from the results, that the non-parametric decision-analysis was consistent with the parametric approach and is considered extremely necessary to verify analysis when the data base is not large enough to support the basic assumptions of normality.

From the data base, using the two-factor analysis of variance, no statistical significance can be attributed to the effects of ambient light and acoustic noise on operator performance. However, considering a third variable, the difference between operators (performance-wise), and incorporating a three-way classification analysis-of-variance, one can conclude that operator performance deteriorates significantly as sound is increased to annoying levels above 80 dBa, while the effect of ambient light had little effect on the number of errors committed. This indicates the adaptability of the operators to *photic* noise.

It is clear also, that the operators' performance was somewhat more critical with the optical display unit. The results show clearly that operator performance deteriorates considerably as ambient light decreases to levels at or below 3 foot-candles of illumination and as acoustic noise increases to levels at or above 80 dBa. Contrary to what one might expect for the teletypewriter terminal, however, the level of ambient light is not significant at all and noise levels above 80 dBa are critical.

Optimal and acceptable environmental conditions for both terminals can be adjudged from tables 1.4 and 1.5. They show the number of errors one can expect for a full range of light and sound conditions for each terminal. The tables are the result of direct computations utilizing the reduced non-linear models (see section 1.5.1 and 1.5.2) which best describe the data.

The experiment, inferences, and conclusions outlined in the preceding sections could be strengthened to give more accurate results by incorporating the following improvements: first, there were the environmental factors which were limited to ambient light and acoustic noise, which may not (in the ranges specified) be the most important factors influencing operator performance. Certainly, operator performance is a function of other stress

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TABLE 1.6 SUMMARY OF ANALYSIS

TYPE OF ANALYSIS AND NULL HYPOTHESIS	DECISION AT THE $\alpha = .05$ LEVEL			
	All Subjects		Trainee Deleted	
	Teletype	Optical	Teletype	Optical
CONTINGENCY TABLE ANALYSIS				
H_0: Factor A Independent of B	Reject	Accept	Accept	Accept
Factor A Alone (Ambient Light)				
H_0: Row Sums are Equal	Reject	Accept	Accept	Reject
H_0: $P_1. = P_4.$	Reject	--	--	Reject
H_0: $P_1. = P_3.$	Reject	--	--	Accept
H_0: $P_1. = P_2.$	Accept	--	--	--
Factor B Alone (Acoustic Noise)				
H_0: Column Sums are Equal	Reject	Reject	Reject	Reject
H_0: $P_{.1} = P_{.4}$	Reject	Reject	Reject	Reject
H_0: $P_{.1} = P_{.3}$	Accept	Accept	Accept	Reject
H_0: $P_{.1} = P_{.2}$	--	--	--	Accept
TWO-FACTOR ANALYSIS OF VARIANCE				
Interaction, H_0: $\gamma_{ij} = 0$			Accept	Accept
Acoustic Noise, H_0: $\beta_j = 0$			Accept	Accept
Ambient Light, H_0: $\alpha_i = 0$			Accept	Accept
THREE-FACTOR ANALYSIS OF VARIANCE				
Interactions:				
H_0: $(\alpha\beta)_{ij} = 0$			Accept	Accept
H_0: $(\alpha\gamma)_{ik} = 0$			Accept	Accept
H_0: $(\beta\gamma)_{jk} = 0$			Accept	Accept
Main Effects:				
Acoustic Noise, H_0: $\alpha_i = 0$			Reject	Reject
Ambient Light, H_0: $\beta_j = 0$			Accept	Accept
Operators, H_0: $\gamma_k = 0$			Reject	Reject

variables, such as training, temperature, humidity, room configuration, fatigue, message backlog, and even the number of people present in the room, to name a few. Additional stress should, therefore, be created through longer messages (fatigue, training), and by imposing a message backlog.

Secondly, by increasing the number of environmental levels, and also the range of levels, the sample size (number of cells) would be increased and thereby provide additional information on the operators' performance. Intuitively, if ambient light became extremely low, i.e., factor $A \leq 1$ foot-candle of illumination, extremely noisy conditions should distract the terminal operator to a much higher degree than indicated here, thus causing a greater number of errors to be committed. Since human operators all behave in a random manner, and no two are alike, the increased ranges and levels should indicate the tolerant stress level one can expect under deteriorating environmental conditions. In this case especially, the models obtained would be strengthened, producing a more accurate reduced model. Consequently, the inference derived from the models would also be strengthened.

Realizing these improvements would, of course, increase the expense of the experimentation; it is felt that the improved results would justify any additional expense. Lastly, from the operations research analyst's point of view, there is a need for further information regarding the proficiency and training of the operators involved in the testing. It was felt that further information could have been acquired had the experimentor known to what degree the subjects *were* random, or if their performance was truly indicative of the general class of communications operators which would operate this type of equipment.

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2. TIME-SERIES MODELING OF MAN/MACHINE INTERFACES

2.1 INTRODUCTION

The best non-linear regression model which characterizes the relation-
ship between the dependent and independent variables found in Section 1,
dealt with the prediction of the number of committed errors as a function of
ambient light and acoustic noise. More often, the communications engineer
is interested in such factors as reliability, performance, and efficiency as
a function of time. Thus, utilizing time-series models, it may be possible
to characterize each operator, or take the group as a whole for predicting
in near-real-time the number of committed errors at times t_1, t_2, t_3, ...,
t_n, in the future. The time-series approach for this type of information is
somewhat unique in that this is one of the first attempts to implement this
methodology in analyzing time-dependent man-machine interface data. In view
of this uniqueness, there are a number of shortcomings that were experienced.
One of the most serious limitations was the sample size. However, enough
information is available so that one can *initiate* the time-series methodology
into this particular subject area. This approach is extremely useful because
it characterizes, within reason, the error performance of any communications
terminal equipment operator working in a tactical environment.

Incorporated into the design of the experiment (detailed in Section 1)
was a four-second time interval counter. This provided a running count of
the number of transcribed errors in each four-second time period for the
duration of the test. Thus, thirty-two non-deterministic time-series were
created (sixteen per terminal, one corresponding to each combination of
environmental factors). Of the time series so obtained, and because of the
magnitude of the work involved, the two most critical environmental combina-
tions are presented, namely, {1,4} and {4,4} (refer to section 1.2). Recall
that criticality was determined by the degree of non-stationarity of the
series, or in other words, the amount of filtering required to bring the
process into statistical equilibrium. Having these stochastic realizations,
we shall proceed to analyze the data in accordance with the recommended
approach of Section 2.2.

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In section 2.3, the appropriate forecasting models will be developed to predict the number of errors committed by the operators. This section will include filtering the data to eliminate non-stationarities, model identification, estimation of the appropriate parameters, and diagnostic checking. Actual "ℓ" steps ahead forecasting and updating the future values of operator performance will be addressed in section 2.4. Finally, a summary and conclusions are presented in section 2.5.

2.2   SOME BASIC CONCEPTS OF TIME-SERIES MODELING

Identifying the Stochastic Realization

In a given physical situation such as the man-machine interface experiment, we have available a time series, say $x_1, x_2, \ldots, x_n$, of n observations. Our aim is to obtain a suitable difference equation or model that will accurately represent the true underlying process which generated the information, x_t, t=1,2,...,n. First, we must identify whether the series x_t exhibits stationary or non-stationary properties. That is, when we speak of stationary time series, we imply that the statistical properties of the series are independent of absolute time. A graphical representation of the experimental data would be of some aid in exercising judgment about the behavior of the data. Of greater importance is the sample autocorrelation function given by:

$$r_{xx}(k) = \frac{c_{xx}(k)}{c_{xx}(0)} \ , \ k = 0,1,2,\ldots,n-1$$

where $c_{xx}(k)$ is the sample autocovariance function defined by:

$$c_{xx}(k) = \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{x}) (x_{t+k} - \bar{x}), \ k=0,1,2,\ldots n-1,$$

of the observed data. If the observations are stationary, the sample autocorrelation function would exhibit fairly rapid dampening. Furthermore, one

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can apply various statistical tests to check for non-stationary properties. We have used Kendall's tau test [7].

If the man-machine interface data were not in statistical equilibrium (stationary), then we can filter out the non-stationary components by using various difference filters. In all environmental combinations of the man-machine interface experiment, the resulting data was shown to contain non-stationary components.

A general difference filter is given by:

$$y_t = (1-B)^d x_t,$$

where B is a shift operator and d is the order of the filter. When d=0, this will indicate that the experimental data is stationary; d=1, will indicate that a first difference filter is necessary to filter the original series, and so on. For the man-machine information, we used filters up to d = 2.

The procedure to determine the proper value for d is to compute the first differences of the original data, x_t, t=1,2,...,n. That is, we process x_t through a first difference filter:

$$y_t = (1-B) x_t = x_t - x_{t-1},$$

which will have (n-1) observations, and then through a second difference filter:

$$w_t = (1-B)^2 x_t = x_t - 2x_{t-1} + x_{t-2}$$

which will have (n-2) observations. For the original observations, x_t, and the filtered y_t and w_t, we calculate the sample autocorrelation function and Kendall's tau [7]. By observing the sample autocorrelation function of the original series, the filtered information, and the results of the trend test (Kendall's tau), we can infer a suitable value for d, that is, the degree of "differencing" necessary to induce the sample autocorrelation function to dampen out fairly rapidly and cause Kendall's tau test not to be significant.

For predicting or forecasting operator performance for one or more time slots in advance, the initial step is to determine the particular process that characterizes our data. There are three basic models that are candidates for this purpose:

a. The Autoregressive Process

b. The Moving Average Process

c. The Mixed Autoregressive-Moving Average Process.

A discrete m-order autoregressive (AR) model is of the form:

$$x_t - \mu = \alpha_1 (x_{t-1} - \mu) + \alpha_2(x_{t-2} - \mu) + \ldots + \alpha_m(x_{t-m} - \mu) + Z_t \quad (2.2.1)$$

where x_t is the autoregressive series which is being generated by the series Z_t, a purely random process, α_1, α_2, ..., α_m are the parameters of the non-ordered process, and μ is the expected value of the series. Such a process assumes that the current value, x_t, of the data has resulted from a linear sum of past values of the series, together with an independent error term, Z_t, not connected with the past.

A discrete q-order moving average (MA) process is given by:

$$x_t - \mu = Z_t - \beta_1 Z_{t-1} - \beta_2 Z_{t-2} - \ldots - \beta_q Z_{t-q} . \quad (2.2.2)$$

This process is a weighted sum of a random series, Z_t. Each realization (committed error, x_t) is made linearly dependent on a Z_t and on one or more previous Z's. Also, μ is the expected value of x_t, and β_1, β_2, ..., β_q are the parameters of the model.

The mixed model consists of the autoregressive and moving average model where m is independent of q. (m is the order of the AR process and q is the order of the MA process).

We shall discuss in some detail a procedural approach in fitting an autoregressive model to the man-machine interface series. A similar approach

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can be followed to formulate the procedure for the moving average and mixed process with minor changes.

The Autoregressive Process

The autoregressive model previously defined can be adapted to represent the characterization of operator performance for the purpose of forecasting. Discussing the theory of the general m^{th} order model is quite complicated and, therefore, we shall first give a brief discussion of the second order model which is quite useful in many physical situations.

The second order discrete autoregressive process may be written as:

$$x_t - \mu = \alpha_1 (x_{t-1} - \mu) + \alpha_2 (x_{t-2} - \mu) + Z_t. \qquad (2.2.3)$$

If, at the initial stage, it was necessary to filter the data to have the information in statistical equilibrium, then we must place certain restrictions on estimating the parameters of the model to make sure that our series remains stationary. To obtain these restrictions on the parameters, we use the concept of Z -transforms, [14], to obtain the characteristic equation of the process. Solving the characteristic equation, we can place conditions on its roots so that the filtered model will not violate the assumption of stationarity.

The Z -transform of equation (2.2.3) is given by:

$$(1 - \alpha_1 Z^{-1} - \alpha_2 Z^{-2}) (x_t - \mu) = Z_t ,$$

and its transfer function $H(Z^{-1})$ is given by:

$$H(Z^{-1}) = \frac{1}{(1 - \alpha_1 Z^{-1} - \alpha_2 Z^{-2})} \qquad (2.2.4)$$

Thus, the characteristic equation of the second-order autoregressive model is:

$$\beta^2 - \alpha_1 \beta - \alpha_2 = 0$$

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whose roots are given by:

$$\zeta_1 = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \text{ and } \zeta_2 = \frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\alpha_2}}{2} .$$

In order for the second-order model to be stationary, we must restrict the estimates of the parameters α_1 and α_2 so that the roots of equation (2.2.4) will be contained within a unit circle, that is, $|\zeta_1|$ and $|\zeta_2|$ must be less than one. This is equivalent to having α_1 and α_2 lie in a triangular region formed by $\alpha_1 + \alpha_2 < 1$, $\alpha_2 - \alpha_1 < 1$, and $-1 < \alpha_1 < 1$. For additional details see [15], and [16].

A similar approach can be carried out by considering models of higher order. The z-transform of the mth order autoregressive (2.2.1) process is given by:

$$(1 - \alpha_1 z^{-1} - \alpha_2 z^{-2} - \ldots - \alpha_m z^{-m})(x_t - \mu) = Z_t. \tag{2.2.5}$$

The transfer function of (2.2.5) is of the form:

$$H(z^{-1}) = \frac{1}{(1 - \alpha_1 z^{-1} - \alpha_2 z^{-2} - \ldots - \alpha_m z^{-m})} . \tag{2.2.6}$$

The characteristic equation of the mth order process is given by:

$$\psi^m - \alpha_2 \psi^{m-1} - \alpha_{-2} \psi^{m-2} - \ldots - \alpha_m = 0 . \tag{2.2.7}$$

Thus, for the general finite autoregressive model to be in statistical equilibrium, we must estimate the parameters of the process so that the roots of equation (2.2.7) must lie within a unit circle.

The Fitting Procedure

The initial stage in developing any one of the three models under consideration usually involves deciding the order, m, of the model, and then, given m, estimating the parameters, μ, α_1, α_2, ..., α_m.

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The criterion for selecting the best order which characterizes the observed series is based upon the residual variance. We proceed by estimating the parameters of the model for different orders, and then the residual variances are computed and plotted against the order of the process. The minimum residual variance will correspond to the order of the model which best describes the experimental data. Thus, for the autoregressive process, it is necessary to first study the estimation of the parameters of the model.

To estimate the parameters of this process, we can use the method of maximum likelihood. We assume that the Z_t process is normal. Then, for a fixed m, the joint probability density function of the variates, Z_{m+1}, Z_{m+2}, ..., Z_m is given by:

$$f_{m+1,\ldots,n}(Z_{m+1},Z_{m+2},\ldots,Z_n) = \frac{1}{(\sqrt{2\pi}\,\sigma_z)^{\frac{n-m}{2}}} e^{-\frac{1}{2\sigma_z^2}\sum_{t=n+1}^{n} Z_t^2} \,,$$

when the expected value of Z_t is zero, and its variance is σ_z^2. Changing from the Z variables to the X variables, according to equation (2.2.1), we have:

$$f_{m+1,\ldots,n}(x_{m+1},x_{m+2},\ldots,x_n \mid x_1, x_2,\ldots,x_m)$$

$$= \frac{1}{(\sqrt{2\pi}\,\sigma_z)^{\frac{n-m}{2}}} e^{-\frac{1}{2\sigma_z^2}\sum_{t=m+1}^{n} \{(x_t-\mu)-\alpha_2(x_{t-1}-\mu)-\ldots-\alpha_m(x_{t-m}-\mu)\}^2} \tag{2.2.8}$$

which is the joint probability density function of $x_{m-1},\ldots,x_n$ conditional on $X_1=x_1$, $X_2=x_2,\ldots$, $X_m=x_m$. Thus, to obtain the joint probability density function of X_1, X_2, ..., X_n, it is only necessary to multiply equation (2.2.8) with the density of X_1, X_2, ..., X_m. Since m, for practical applications, is usually small, the net effect of not carrying out this multiplication is small and will be omitted. For details, see [16].

The Log-likelihood function of the process may be written as follows:

$$L(\mu,\alpha_1,\alpha_2,\ldots,\alpha_m \mid x_1,x_2,\ldots,x_m)=-(n-m)\ln\sqrt{2\pi}-(n-m)\ln\sigma_z$$

$$-\frac{1}{2\sigma_z^2}\sum_{t=m+1}^{n}\{(x_t-\mu)-\alpha_1(x_{t-1}-\mu)-\ldots-\alpha_m(x_{t-m}-\mu)\}^2 . \qquad (2.2.9)$$

The sum-of-squares function, given by:

$$S(\mu,\alpha_1,\alpha_2,\ldots,_m\mid x_1,x_2,\ldots,x_m)$$

$$=\sum_{t=m+1}^{n}\{(x_t-\mu)-\alpha_1(x_{t-1}-\mu)-\ldots-\alpha_m(x_{t-m}-\mu)\}^2 , \qquad (2.2.10)$$

is needed to estimate the parameters of the model. Differentiating equation (2.2.10), with respect to μ, α_1, α_2, ..., α_m, setting them equal to zero, and solving the m+1 system of equations, we can obtain their maximum likelihood estimates.

For the second-order autoregressive process, we differentiate equation (2.2.10) with respect to μ, α_1, and α_2, and obtain the following normal equations:

a. $(\bar{x}_3-\hat{\mu}) = \hat{\alpha}_1(\bar{x}_2-\hat{\mu}) + \hat{\alpha}_2(\bar{x}_1-\hat{\mu})$,

b. $\sum_{t=3}^{n}(x_t-\hat{\mu})(x_{t-1}-\hat{\mu})=\hat{\alpha}_1\sum_{t=3}^{n}(x_{t-1}-\hat{\mu})^2+\hat{\alpha}_2\sum_{t=3}^{n}(x_{t-1}-\hat{\mu})(x_{t-2}-\hat{\mu})$,

and

c. $\sum_{t=3}^{n}(x_t-\hat{\mu})(x_{t-2}-\hat{\mu})=\hat{\alpha}_1\sum_{t=3}^{n}(x_{t-1}-\hat{\mu})(x_{t-2}-\hat{\mu})+\hat{\alpha}_2\sum_{t=3}^{n}(x_{t-2}-\hat{\mu})^2$,

where

$$\bar{x}_j = \frac{1}{n-3}\sum_{t=3}^{n}x_{t-3+j} , \quad j=1,2,3 .$$

Since $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_3$ are usually close to the overall mean, $\bar{x}$, we can use it as an approximate estimate of μ. Furthermore, we can obtain good approximate estimates of (a) through (c) using the sample autocorrelation function at lag

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one, $r_{xx}(1)$. That is,

$$c_{xx}(1) \simeq \hat{\alpha}_1 \, c_{xx}(0) + \hat{\alpha}_2 \, c_{xx}(1) \,,$$

and
$$c_{xx}(2) \simeq \hat{\alpha}_1 \, c_{xx}(1) + \hat{\alpha}_2 \, c_{xx}(0) \,.$$
$$\} \quad (2.2.11)$$

The autocovariance is an even function, thus, we can write equation (2.2.11) as follows:

$$c_{xx}(j) \simeq \hat{\alpha}_1 \, c_{xx}(j-1) + \hat{\alpha}_2 \, c_{xx}(j-2) \,, \quad j = 1,2 \,. \qquad (2.2.12)$$

An approximate estimate of the parameters α_1 and α_2 is given by:

$$\hat{\alpha}_1 \simeq \frac{r_{xx}(1)[1-r_{xx}(2)]}{1-r_{xx}^2(1)} \,,$$

and
$$\hat{\alpha}_2 \simeq \frac{r_{xx}(2)-r_{xx}^2(1)}{1-r_{xx}^2(1)} \,.$$
$$\} \quad (2.2.13)$$

Also, an estimate of the residual sum of squares can be obtained in terms of the sample autocovariance function. That is:

$$S(\hat{\mu},\hat{\alpha}_1,\hat{\alpha}_2) \simeq (n-2) \, \{c_{xx}(0)-\hat{\alpha}_1 c_{xx}(1)-\hat{\alpha}_2 c_{xx}(2)\} \,, \qquad (2.2.14)$$

and the residual variance of Z_t is given by:

$$S_z^2 = \frac{1}{n-5} \, S(\hat{\mu}, \, \hat{\alpha}_1, \, \hat{\alpha}_2).$$

Similar expressions can be obtained for estimating the parameters for the general finite autoregressive model. The normal equations may be approximated by using the sample autocovariance given by:

$$c_{xx}(j) \simeq \hat{\alpha}_1 c_{xx}(j-1) + \hat{\alpha}_2 c_{xx}(j-2) \, \ldots \, + \hat{\alpha}_m c_{xx}(j-m) \qquad (2.2.15)$$

$j = 1, 2, \ldots, m$. Approximate estimates can be obtained for the parameters $\alpha_1, \alpha_2, \ldots, \alpha_m$, by solving the m simultaneous equations (2.2.15).

The residual sum of squares and the residual variance may be obtained by using the following approximations:

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$$S(\hat{\mu}, \hat{\alpha}_1, \ldots, \hat{\alpha}_m) \simeq (n-m) \{c_{xx}(0) - \hat{\alpha}_1 c_{xx}(1) - \ldots - \hat{\alpha}_m c_{xx}(m)\}, \quad (2.2.15a)$$

and

$$S_z^2 \simeq \frac{1}{n-2m-1} \, S(\hat{\mu}, \hat{\alpha}_1, \ldots, \hat{\alpha}_m) , \quad (2.2.15b)$$

respectively.

Checking the Fit of the Model

Once we have selected the best process that characterizes the man-machine interface data and have its parameters estimated, diagnostic checks are made on the model to determine its adequacy. Using this model, we can obtain a series that should simulate the behavior of the original information. If the original information were filtered, that is, d was different from zero, it would now be necessary to use a "backwards filter", replacing y_t in the model with $(1-B)^d x_t$, and using the resulting process to forecast operator performance. For example, if we fitted a first-order autoregressive model:

$$\hat{y}_t - \hat{\mu} = \hat{\alpha}_1 (y_{t-1} - \hat{\mu}) + Z_t , \quad (2.2.16)$$

where $y_t = (1-B)x_t = x_t - x_{t-1}$ is the filter used in the original data, then inserting the filter into (2.2.16), we have:

$$x_t = \varpi_0 + \varpi_1 x_{t-1} + \varpi_2 x_{t-2} + Z_t ,$$

where $\varpi_0 = \hat{\mu}(1-\hat{\alpha}_1)$, $\varpi_1 = (1+\hat{\alpha})$ and $\varpi_2 = -\hat{\alpha}_1$. Thus, the mth order autoregressive process, using a first difference filter, can be written as follows:

$$x_t = \tau_0 + \tau_1 x_{t-1} + \tau_2 x_{t-2} + \ldots + \varpi_{m+d} x_{t-m-d} + Z_t, \quad (2.2.17)$$

where the values of τ_i, i = 1, 2, ..., m+d, will depend on m and d.

For the fitted model to give a good characterization of operator performance, the residuals, $r_t = x_t - \hat{x}_t$, t = 1, 2, ..., n, should behave approximately like random deviates. Hence, the sample autocorrelation function should effectively be zero for all lags except the zeroth lag.

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Forecasting and Updating the Model

One of the aims in having fitted an autoregressive process to the experimental data, is to forecast future values of the committed errors (operator performance). If we wish to forecast a particular piece of information, $x_{t+\ell}$, $\ell \geq 1$, when we are presently at time slot t, then the forecast is made at origin t for a lead-time ℓ. Of course, the shorter the lead-time ℓ, the more accurate our forecasted value will be.

The minimum mean square error forecast for any lead time is given by the conditional expectation, [15], $E_t[X_{t+\ell}]$, of $x_{t+\ell}$, at time slot (origin) t, given knowledge of all x's up to time t. That is:

$$E_t[X_{t+\ell}] = x_t(\ell)$$

Replacing t with $t + \ell$ in equation (2.2.17), we have:

$$x_{t+\ell} = \varpi_0 + \varpi_1 x_{t+\ell} + \varpi_2 x_{t+\ell-2} + \cdots + \varpi_{m+d} x_{t+\ell-m-d} + Z_{t+\ell} \ .$$

The minimum mean square error forecast of the ionospheric data is given by:

$$E_t[X_{t+\ell}] = \varpi_0 + \varpi_1 E_t[X_{t+\ell-1}] + \cdots + \varpi_{m+d} E_t[X_{t+\ell-m-d}] + E_t[Z_{t+\ell}] \ .$$

$$(2.2.18)$$

For j, a non-negative integer, we know, [15], that:

$$E_t[X_{t+j}] = \hat{x}_t(j), \ E_t[Z_{t+j}] = 0, \ j = 1, 2, \ldots , \qquad (2.2.19)$$

and

$$E_t[X_{t-j}] = x_{t-j}, \ E_t[Z_{t-j}] = Z_{t-j} = x_{t-j} - \hat{x}_{t-j-1} \ . \qquad (2.2.20)$$

Therefore, we can write equation (2.2.18) as follows:

$$\hat{x}_t(\ell) = \varpi_0 + \varpi_1 x_{t+\ell-1} + \cdots + \varpi_{m+d} x_{t+\ell-m-d} \ . \qquad (2.2.21)$$

The variance of the ℓ step ahead forecast-error for any time slot t, is the expected value of:

$$e_t^2(\ell) = [X_{t+\ell} - \hat{x}_t(1)]^2 \ . \qquad (2.2.22)$$

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Box and Jenkins, [16], have shown that the variance of the lead time, ℓ, is given by:

$$\text{Var}(\ell) = \{1 + \sum_{j=1}^{\ell-1} \theta_j^2\} \; \sigma_z^2 \qquad (2.2.23)$$

where σ_z^2 is estimated by s_z^2, that is:

$$s_z^2 = \frac{S(\hat{\mu}, \hat{\alpha}_1, \ldots, \hat{\alpha}_m)}{n} \; ,$$

and θ_j is given by:

$$\theta_j = 0, \; j < 0$$

$$\theta_0 = 1$$

$$\theta_1 = \varphi_1$$

$$\theta_2 = \varphi_1 \theta_1 + \varphi_2$$

$$\vdots$$

$$\theta_j = \varphi_1 \theta_{j-1} + \ldots + \varphi_{m+d} \; \theta_{j-m-d} \; . \qquad (2.2.24)$$

The $(1-\alpha)\%$ confidence limits for $x_{t+\ell}$ is given by:

$$\text{Pr}\; \{\hat{x}_t(\ell) - U_{\frac{\alpha}{2}} (1+\sum_{j=1}^{\ell-1} \theta_j^2)^{\frac{1}{2}} \; s_z \leq x_{t+\ell} \leq \hat{x}_t(\ell) + U_{\frac{\alpha}{2}} (1+\sum_{j=1}^{\ell-1} \theta_j^2)^{\frac{1}{2}} s_z \; = 1 - \alpha$$

where $U_{\frac{\alpha}{2}}$ is the deviate from the unit normal probability distribution.

In man-machine interface problems, we are often interested in forecasting future values of an observed series for several time slots in advance. When we forecast values at leads greater than or equal to two ($\ell \geq 2$) with an autoregressive process, the forecasted value will be dependent on previously forecasted values; but, as additional data becomes available, we can update our old forecast by:

$$\hat{x}_{t+1}(\ell) = x_t(\ell+1) + \theta_\ell Z_{t+1} \; .$$

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That is, the "t" origin forecast of $x_{t+\ell+1}$ can be updated to become the "t + 1" origin forecast of the same value, $x_{t+\ell+1}$, by adding a constant multiple of the one-step ahead forecast error Z_{t+1}, where

$$Z_{t+1} = X_{t+1} - \hat{x}_t(\ell)$$

is used with multiplier θ_ℓ.

2.3 FORECASTING MODELS FOR CHARACTERIZING MAN-MACHINE INTERFACES

The objective of this section is to utilize both non-stationary and stationary data, generated by the experiment outlined in section 1.2, to develop models for predicting communications operator performance. More precisely, we shall fit the appropriate model, either AR, MA, or ARMA, to man-machine performance data obtained for two communications terminal equipments. The series involved will be those for a deteriorating environment {1,4} and the most deteriorated environment {4,4} as specified in table 1.1. This involves a detailed analysis of four observed series where the discrete intervals have a duration of four seconds. Thus, we are concerned with the response variable, namely, the number of committed errors per four-second interval, as a function of the transcription time of a message (refer to section 1.2).

2.3.1 Identifying the Series

To determine if the series for the standard teletypewriter terminal (TTY) and the Optical Display Terminal (ODT) exhibit stationary or non-stationary properties, one must first try to visually detect any trend or non-randomness. Figures 2.1, 2.2, 2.3, and 2.4 are plots of the data, x_t, t = 1, 2, ..., n, where n < 50. The visual interpretation of the series appears to indicate, for the environmental combinations {1,4} and {4,4}, that non-stationarities do exist. In all cases, however, this is not true, as we will now proceed to verify. It is to be again emphasized that visual interpretation must be validated by using graphs of the autocorrelation functions and statistical tests for trend.

Figures 2.5, 2.6, 2.7 and 2.8 show the sample autocorrelation functions for the observed data. Figures 2.5 and 2.8 are excellent examples of rapid
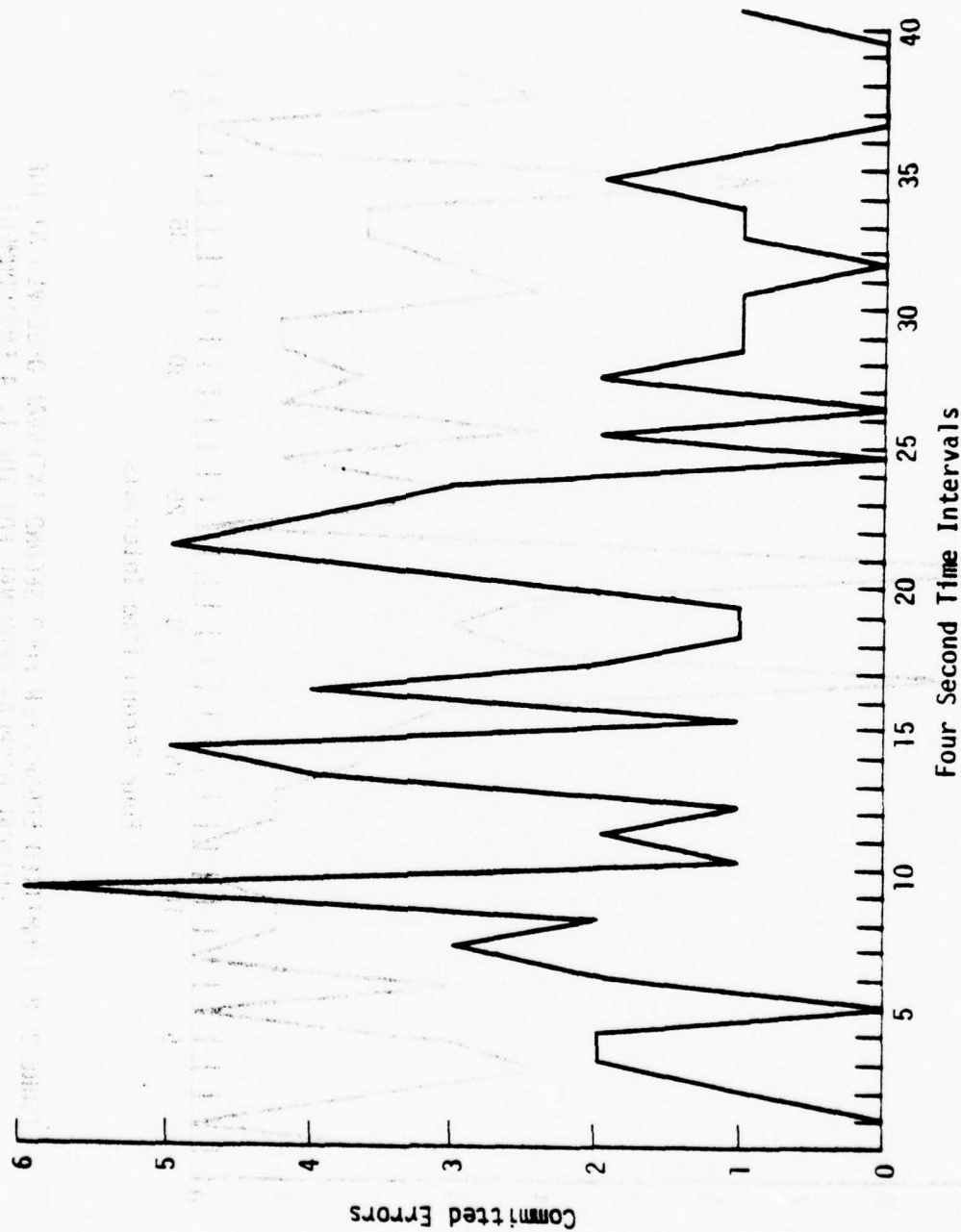
52

FIGURE 2.1 COMMITTED ERRORS PER FOUR SECOND INTERVAL OBSERVED AT THE
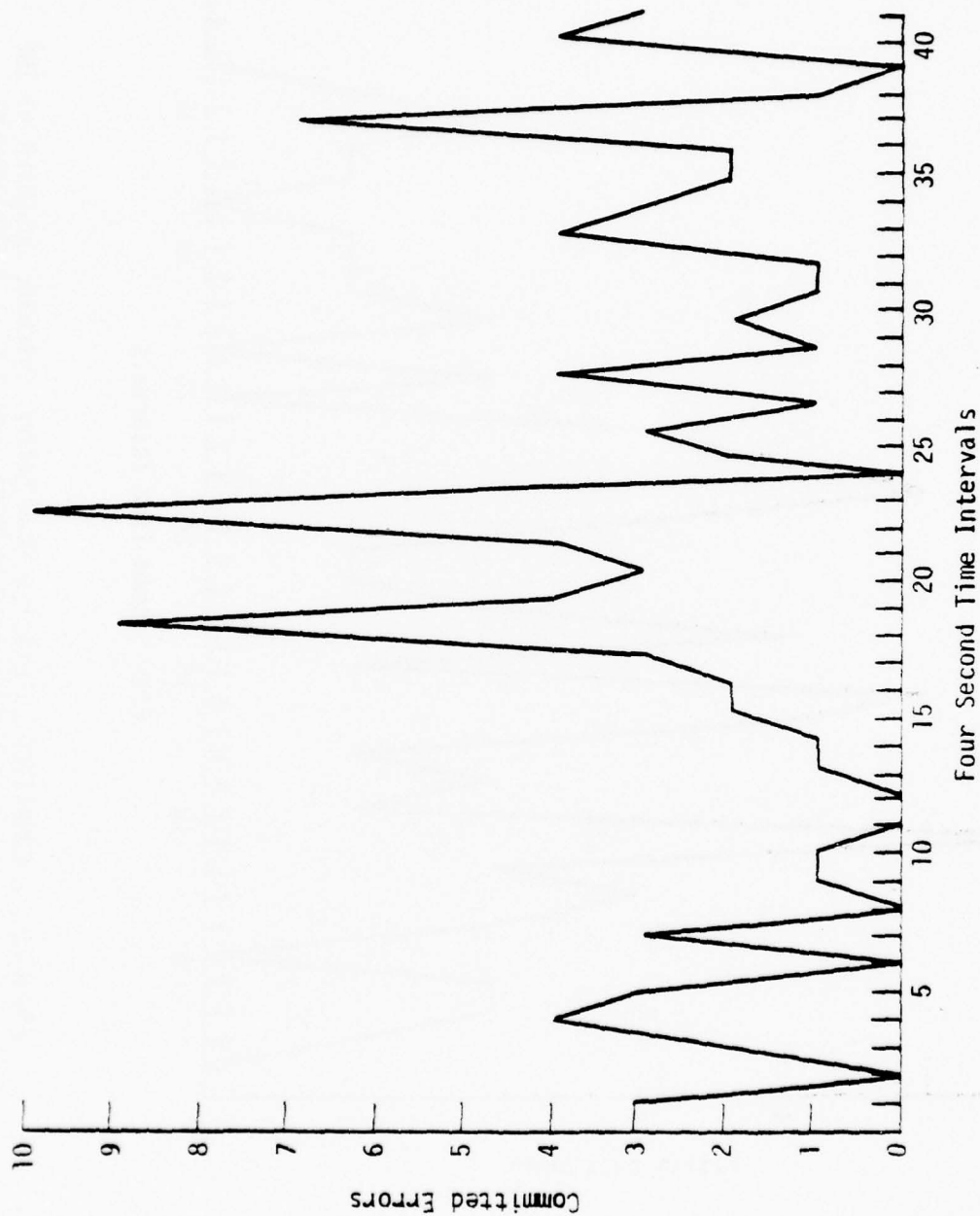TELETYPEWRITER TERMINAL FOR THE 1, 4 ENVIRONMENT

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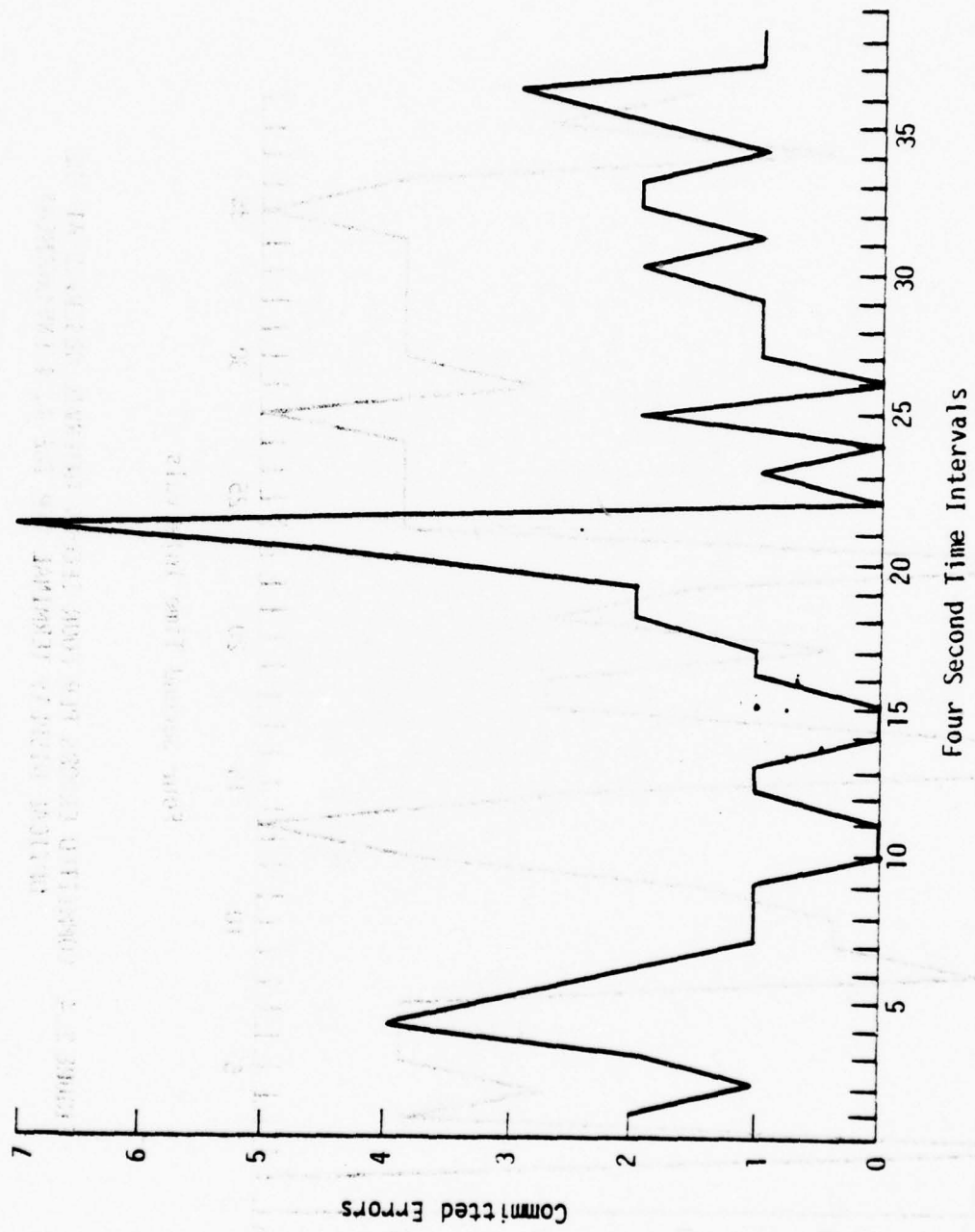FIGURE 2.2 COMMITTED ERRORS PER FOUR SECOND INTERVAL OBSERVED AT THE OPTICAL DISPLAY TERMINAL FOR THE 1, 4 ENVIRONMENT

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FIGURE 2.3 COMMITTED ERRORS PER FOUR SECOND INTERVAL OBSERVED AT THE
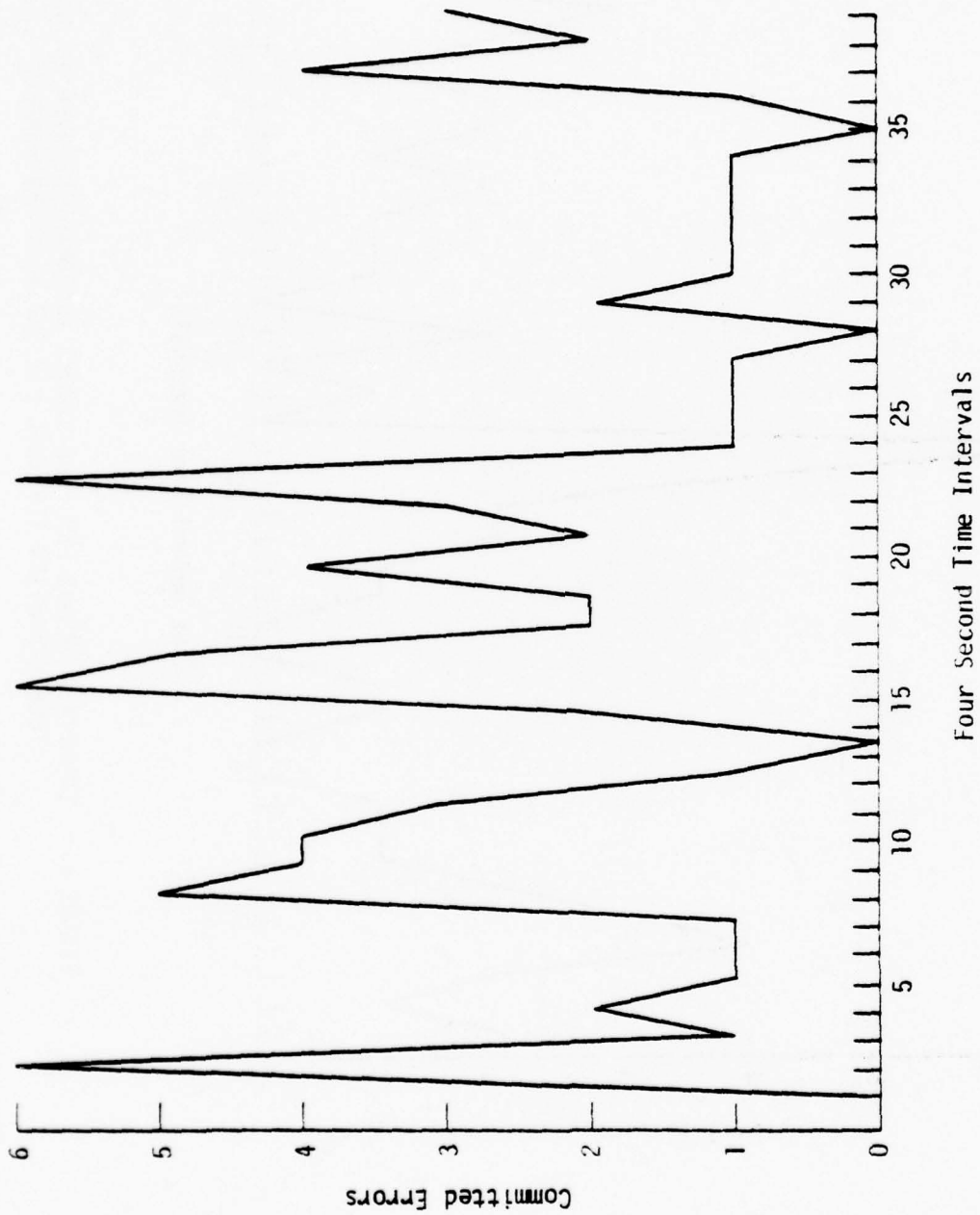TELETYPEWRITER TERMINAL FOR THE 4, 4 ENVIRONMENT

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FIGURE 2.4 COMMITTED ERRORS PER FOUR SECOND INTERVAL OBSERVED AT THE
OPTICAL DISPLAY TERMINAL FOR THE 4, 4 ENVIRONMENT

Four Second Time Intervals

Sample Autocorrelation
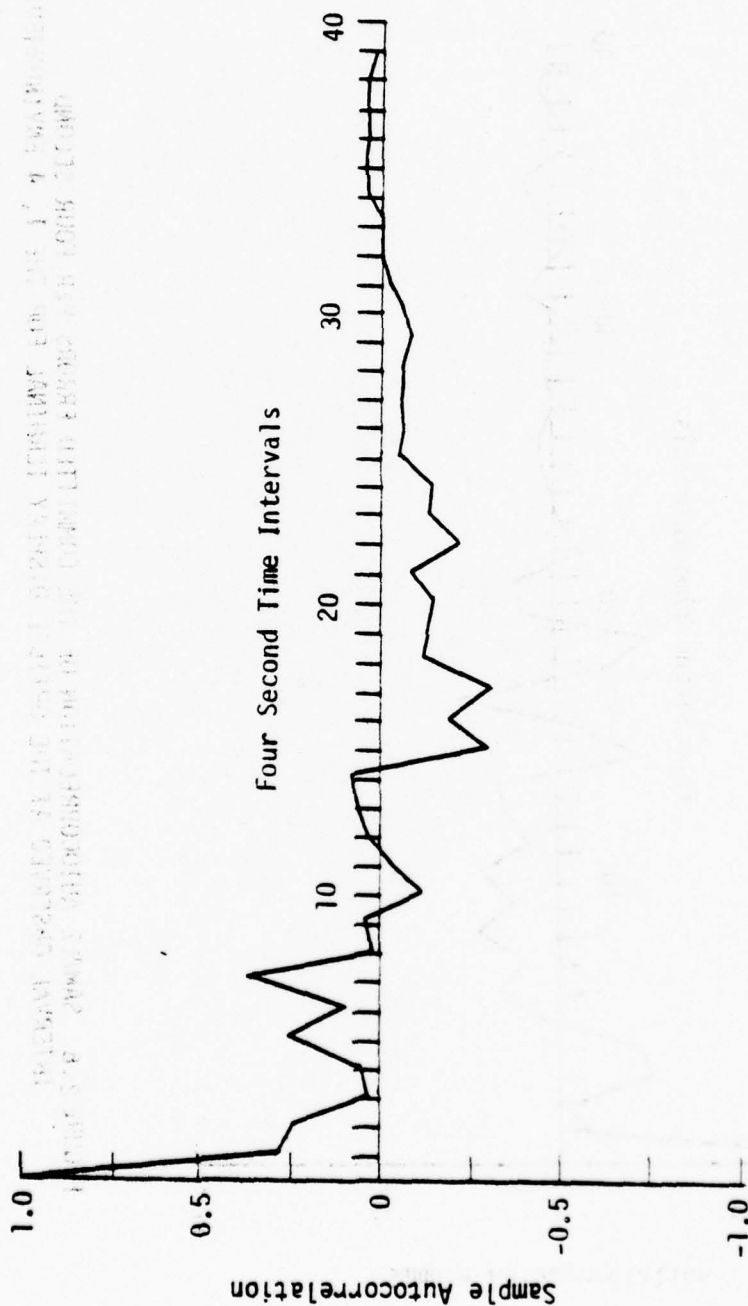
1.0

0.5

0

-0.5

-1.0

10 20 30 40

FIGURE 2.5 SAMPLE AUTOCORRELATION OF THE COMMITTED ERRORS PER FOUR SECOND INTERVAL OBSERVED AT THE TELETYPEWRITER TERMINAL FOR THE 1, 4 ENVIRONMENT
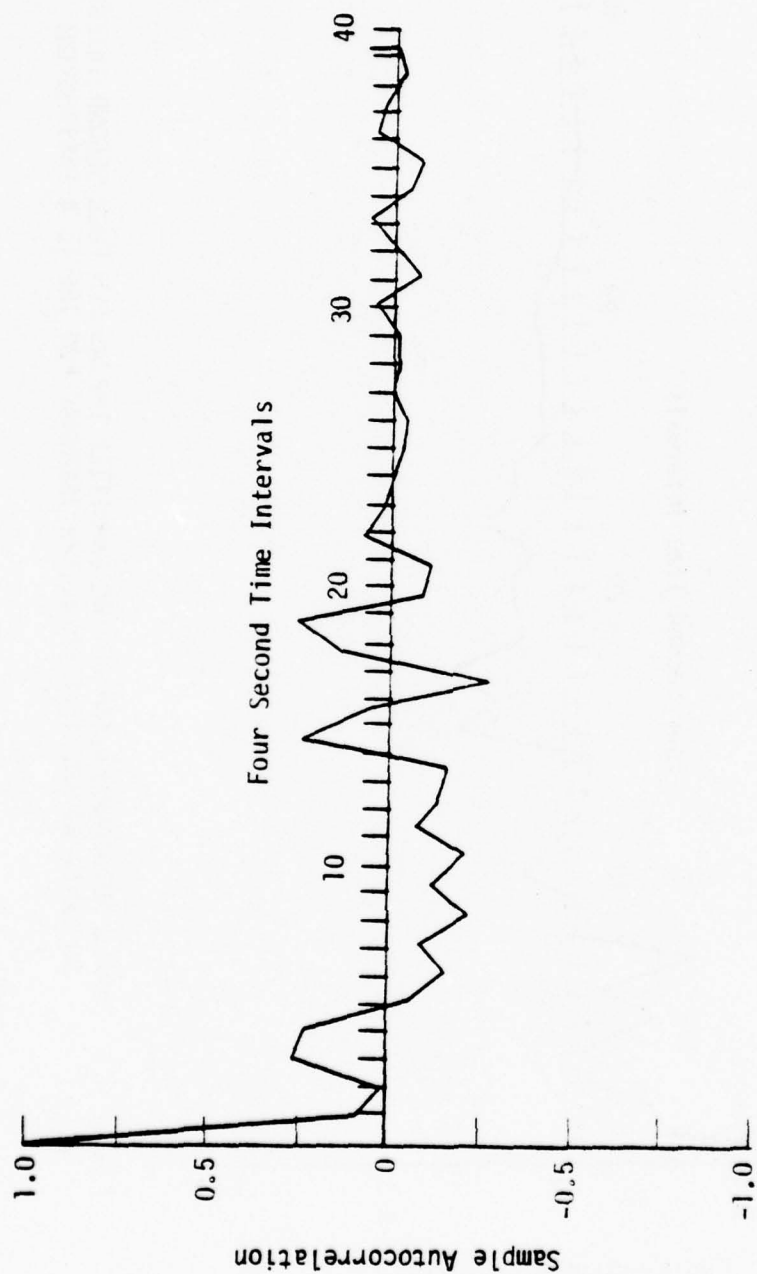
FIGURE 2.6 SAMPLE AUTOCORRELATION OF THE COMMITTED ERRORS PER FOUR SECOND
INTERVAL OBSERVED AT THE OPTICAL DISPLAY TERMINAL FOR THE 1, 4 ENVIRONMENT
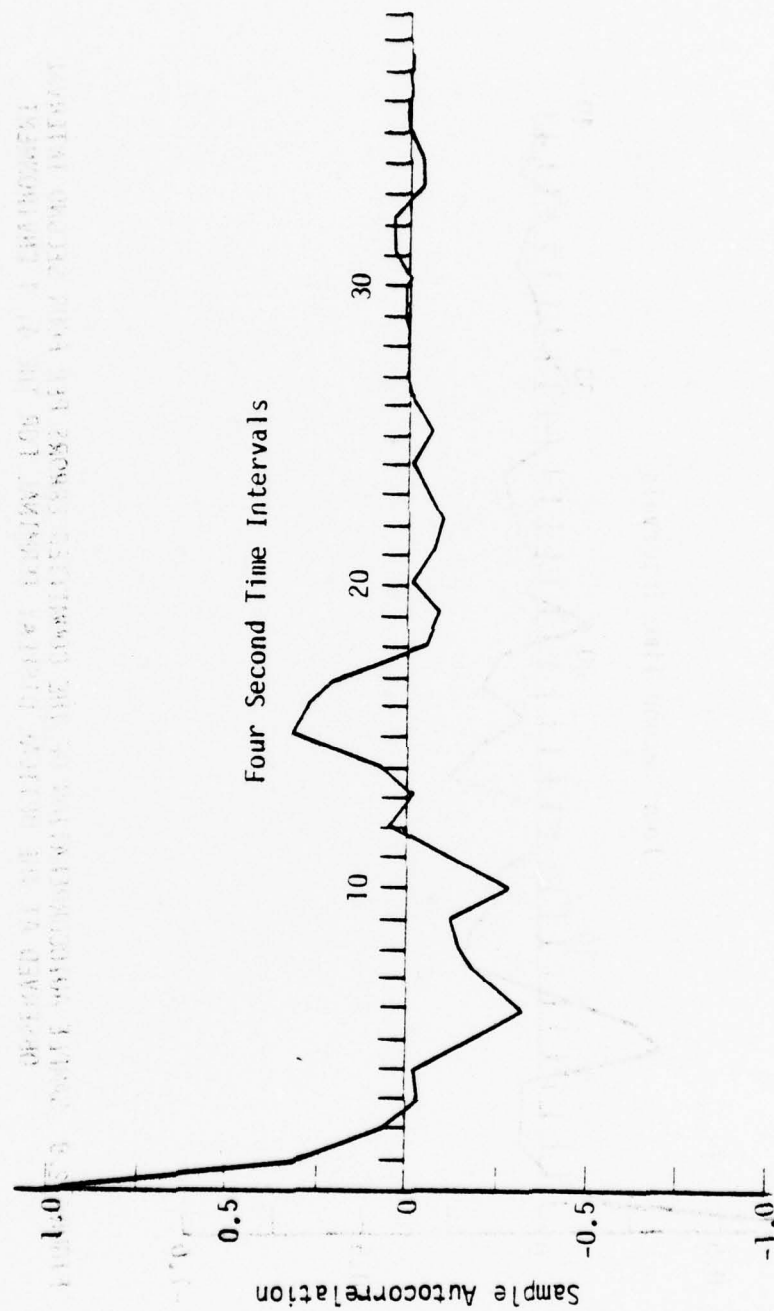
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FIGURE 2.7 SAMPLE AUTOCORRELATION OF THE COMMITTED ERRORS PER FOUR SECOND INTERVAL OBSERVED AT THE TELETYPEWRITER TERMINAL FOR THE 4, 4 ENVIRONMENT
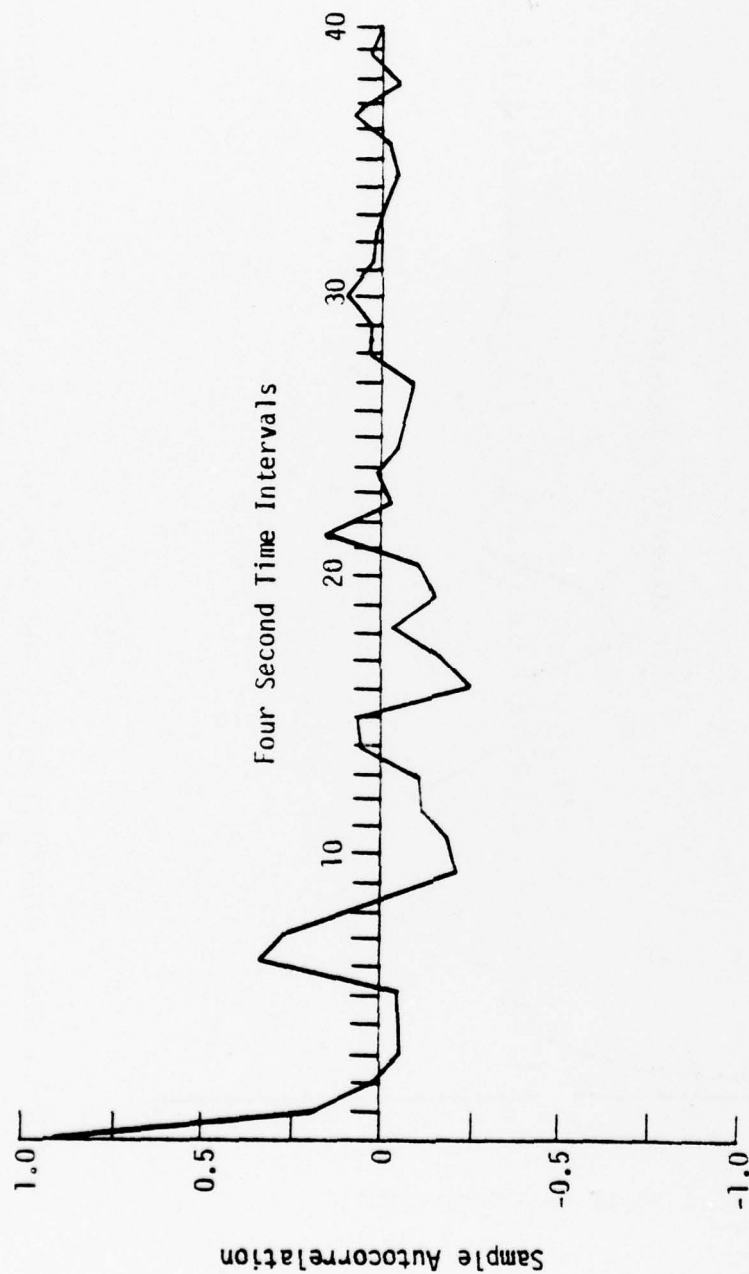
FIGURE 2.8 SAMPLE AUTOCORRELATION OF THE COMMITTED ERRORS PER FOUR SECOND INTERVAL
OBSERVED AT THE OPTICAL DISPLAY TERMINAL FOR THE 4, 4 ENVIRONMENT

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dampening and indicate that the {1,4} environment (TTY data), and the {4,4} environment (ODT data), are stationary realizations requiring no filtering of the information. As to the contrary, figures 2.6 and 2.7 show some dampening, but due to large peaks between time slots 10 through 20, the indication is that some form of filtering is required. Figures 2.9 and 2.10 show the sample autocorrelations of the first filtered information for the {1,4} ODT, and {4,4} TTY. Dampening in these cases has obviously been improved, especially in the case of the {1,4} ODT data. Still, however, large peaks remain in figure 2.10 between the 10th through 20th time slots. With the indication that the {1,4} ODT data is now stationary, the sample autocorrelation of the second differenced {4,4} TTY data was plotted (see figure 2.11). Dampening in this case is again improved about the zero axis due primarily to the increased peak at the second time slot. Now one can surmise that the second differenced data is in statistical equilibrium.

In each case, Kendall's Tau test was performed on the filtered information (the {1,4} ODT and the {4,4} TTY data) and on the unfiltered information (the {1,4} TTY, and the {4,4} ODT data), to verify the indications shown by the graphs of the autocorrelation functions. Table 2.1 shows the results of the trend tests.

Table 2.1 Kendall's Tau Test for the
Man-Machine Interface Series

SERIES	CALCULATED STATISTICS			
	OBSERVED SERIES	FIRST DIFFERENCE DATA	SECOND DIFFERENCE DATA	REFERENCE $Z_{.05}$
{1,4} TTY	-0.520	-----	-----	±1.645
{1,4} ODT	2.656	0.979	-----	±1.645
{4,4} TTY	2.477	2.328	1.089	±1.645
{4,4} ODT	0.817	-----	-----	±1.645

Clearly, in comparing the calculated statistics to the appropriate reference at the α = .05 level of significance, the indications of the sample autocorrelation functions are verified. Therefore, one must conclude:

a. that the {1,4} ODT data is stationary after a first difference filter is used,
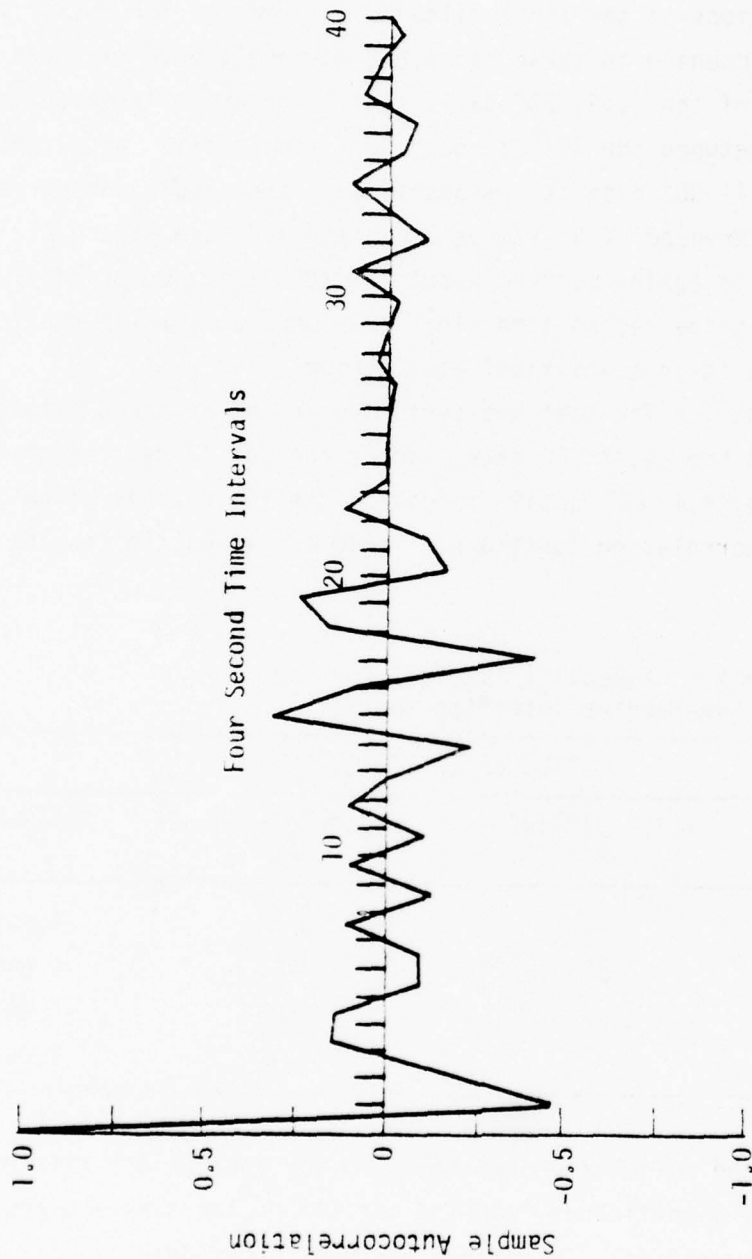
61

FIGURE 2.9 SAMPLE AUTOCORRELATION OF THE FIRST DIFFERENCE DATA OF THE OPTICAL DISPLAY TERMINAL FOR THE 1, 4 ENVIRONMENT
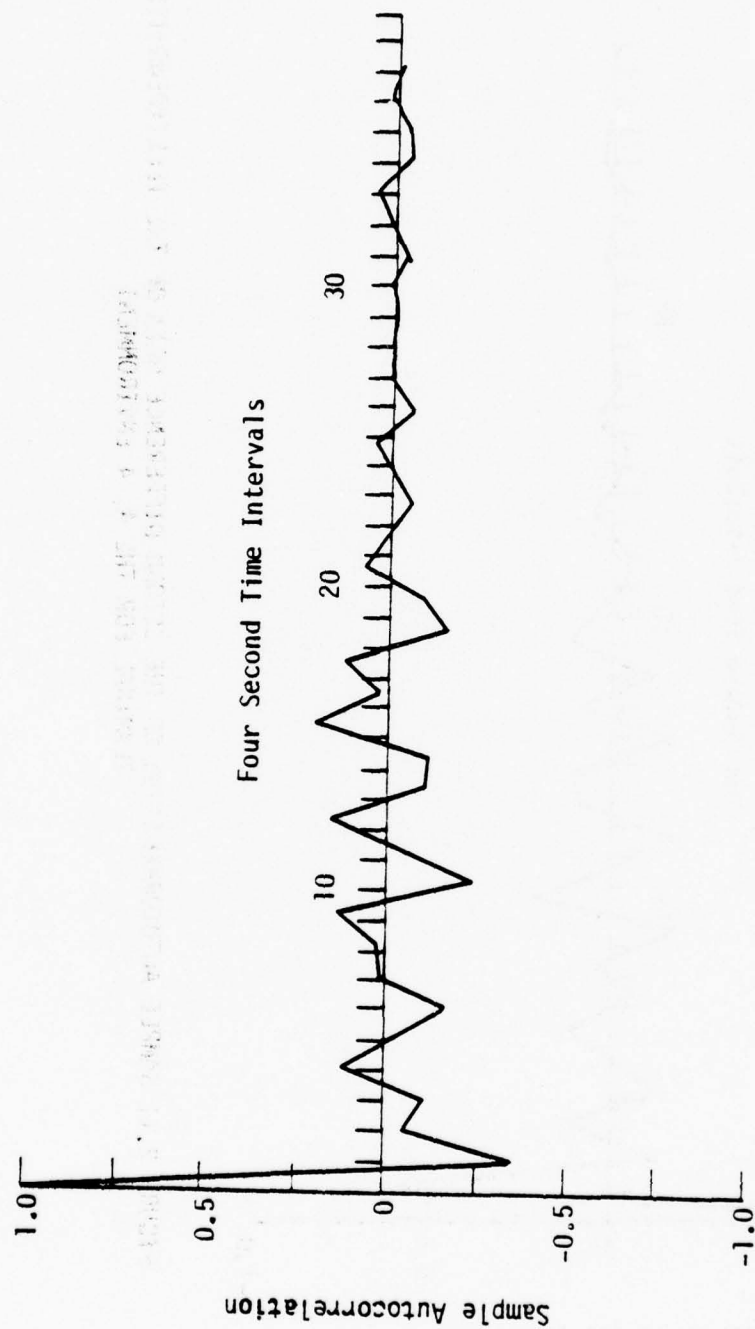
62

FIGURE 2.10 SAMPLE AUTOCORRELATION OF THE FIRST DIFFERENCE DATA OF THE
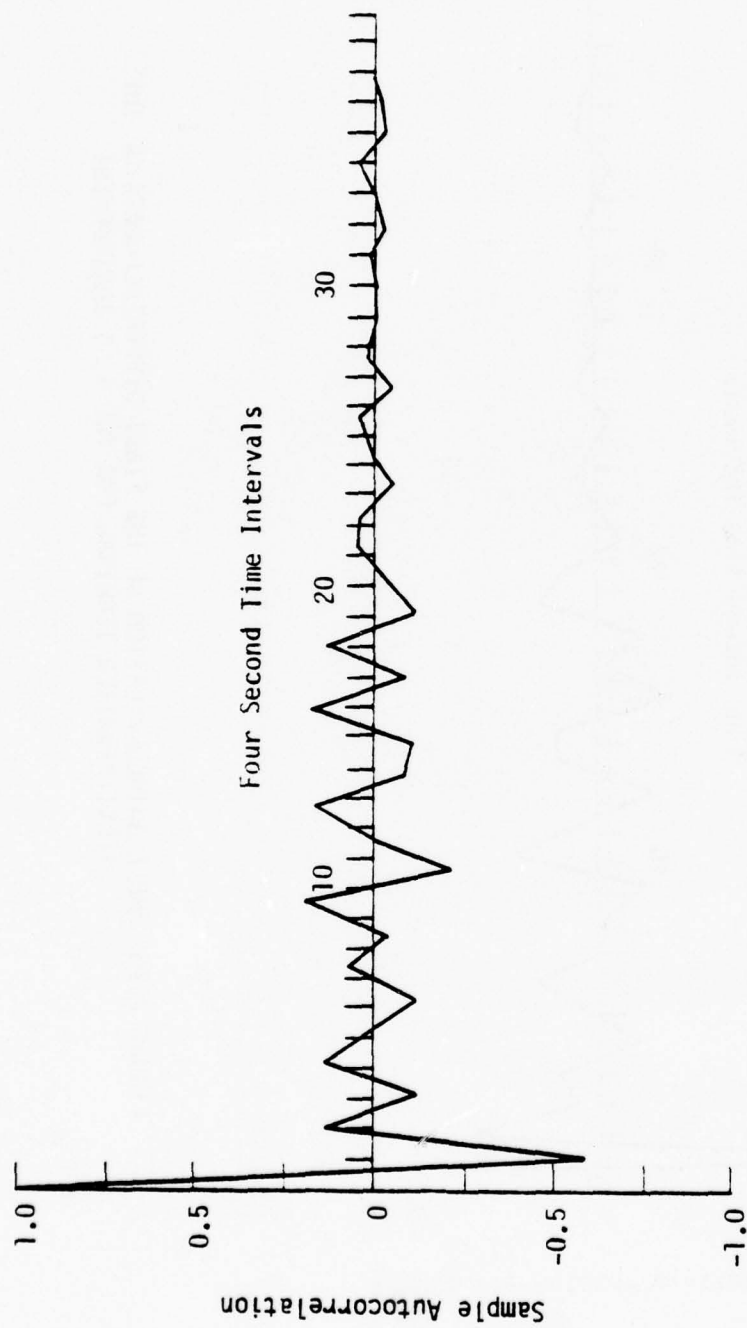TELETYPEWRITER TERMINAL FOR THE 4, 4 ENVIRONMENT

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FIGURE 2.11 SAMPLE AUTOCORRELATION OF THE SECOND DIFFERENCE DATA OF THE TELETYPEWRITER TERMINAL FOR THE 4, 4 ENVIRONMENT

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b. that the {1,4} TTY observed data and the {4,4} ODT observed data are in statistical equilibrium and no filtering is required, and,

c. that the {4,4} TTY data requires a second difference filter to remove the non-stationarities.

2.3.2 Fitting the Models for Man-Machine Interface Data

To fit one of the stationary stochastic models discussed, namely, the autoregressive, moving average, and a mixture of the two, the parameters are estimated for various orders (m,q), taking into consideration the restrictions on the parameters to insure stationarity and/or the invertibility of the stochastic process. Simultaneously, the residual variances are computed for each of the orders under consideration. Figures 2.12, 2.13, 2.14, and 2.15 show the calculated residual variances as a function of order, (m,q). We use the criterion of minimum residual variance to select the appropriate model. If there are competitive residual variances, then we select the model of least order to avoid complicated handling of the data. This is especially important for field implementation of the forecasting scheme which can be accomplished with a hand-held calculator, given the appropriate parameter values. Thus, from the above figures, the following models were selected:

i) (2,3) ARMA process for the teletypewriter terminal, {1,4} environment,

ii) (3,0) AR process for the optical display terminal, {1,4} environment,

iii) (1,3) ARMA process for the teletypewriter terminal, {4,4} environment, and

iv) (0,3) MA process for the optical display terminal, {4,4} environment.

In subsequent sections, it will be shown that the above model identifications are the most appropriate ones to characterize the data.

With the order of the process determined, the associated parameters which were simultaneously computed are now also known. Table 2.2 shows these estimates for the appropriately filtered series.

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FIGURE 2.12 MODEL ORDER vs. RESIDUAL VARIANCE FOR THE MAN/MACHINE INTERFACE DATA, TELETYPEWRITER TERMINAL, 1,4 ENVIRONMENT

FIGURE 2.13 MODEL ORDER vs. RESIDUAL VARIANCE FOR THE MAN/MACHINE INTERFACE DATA, OPTICAL DISPLAY TERMINAL, 1,4 ENVIRONMENT
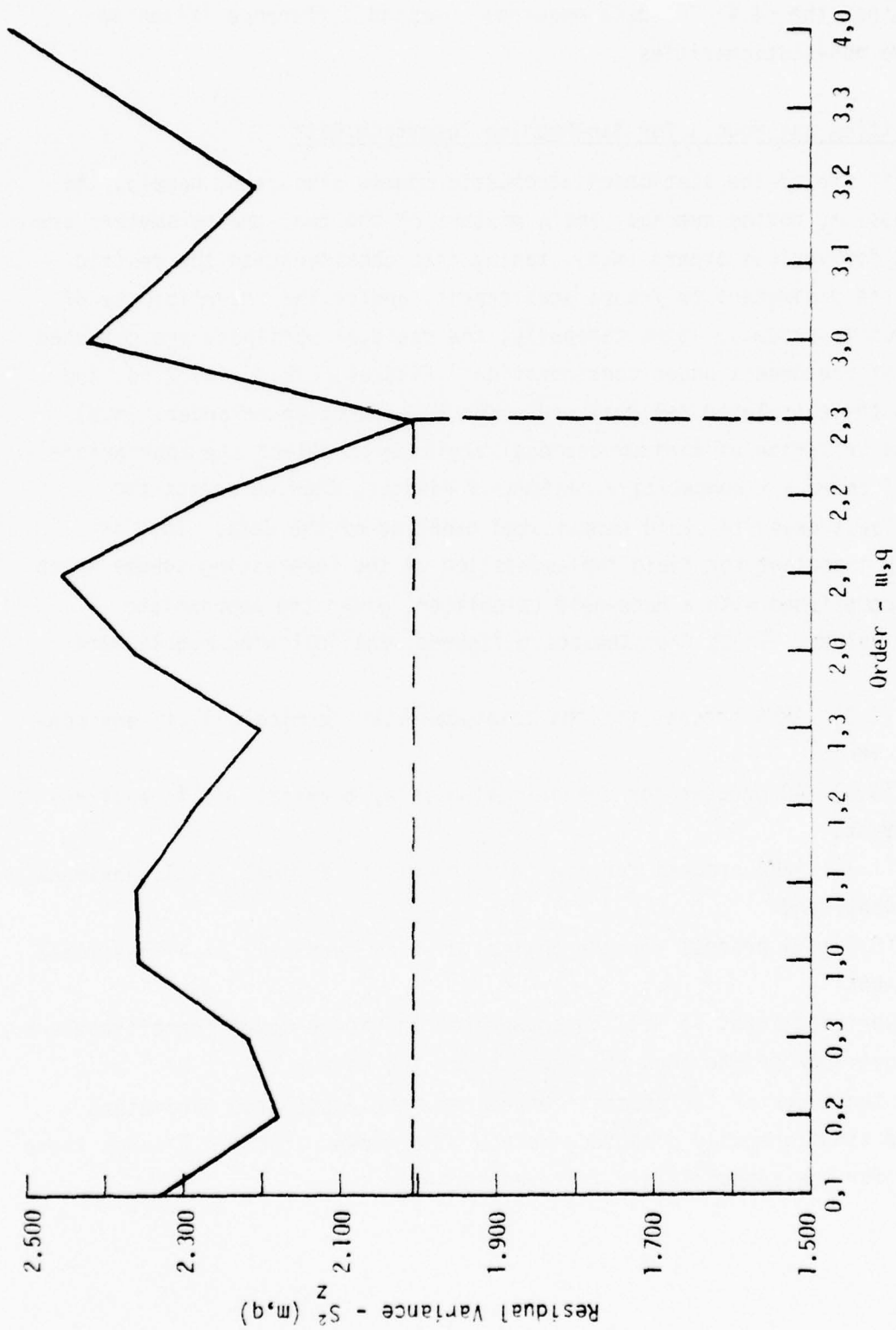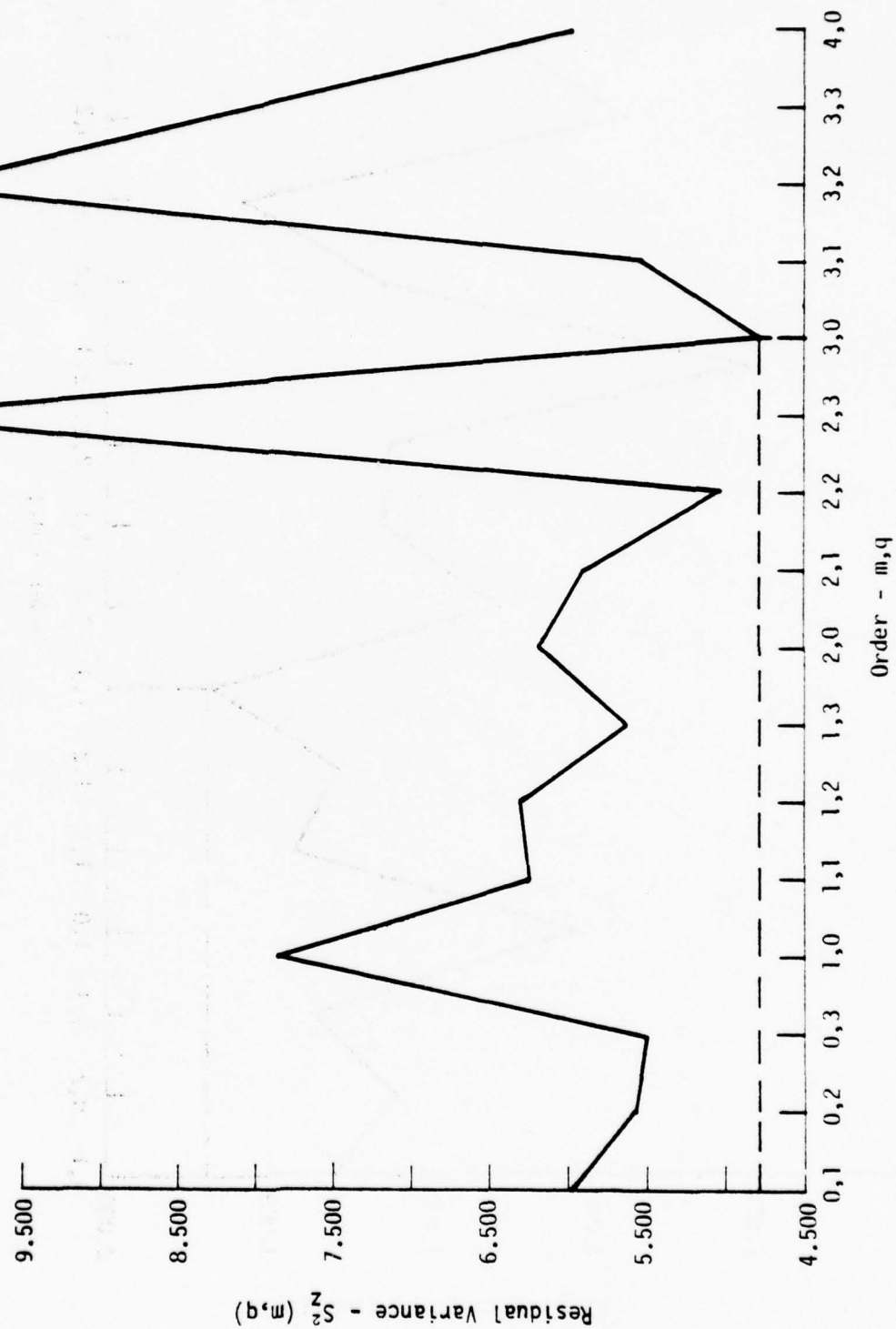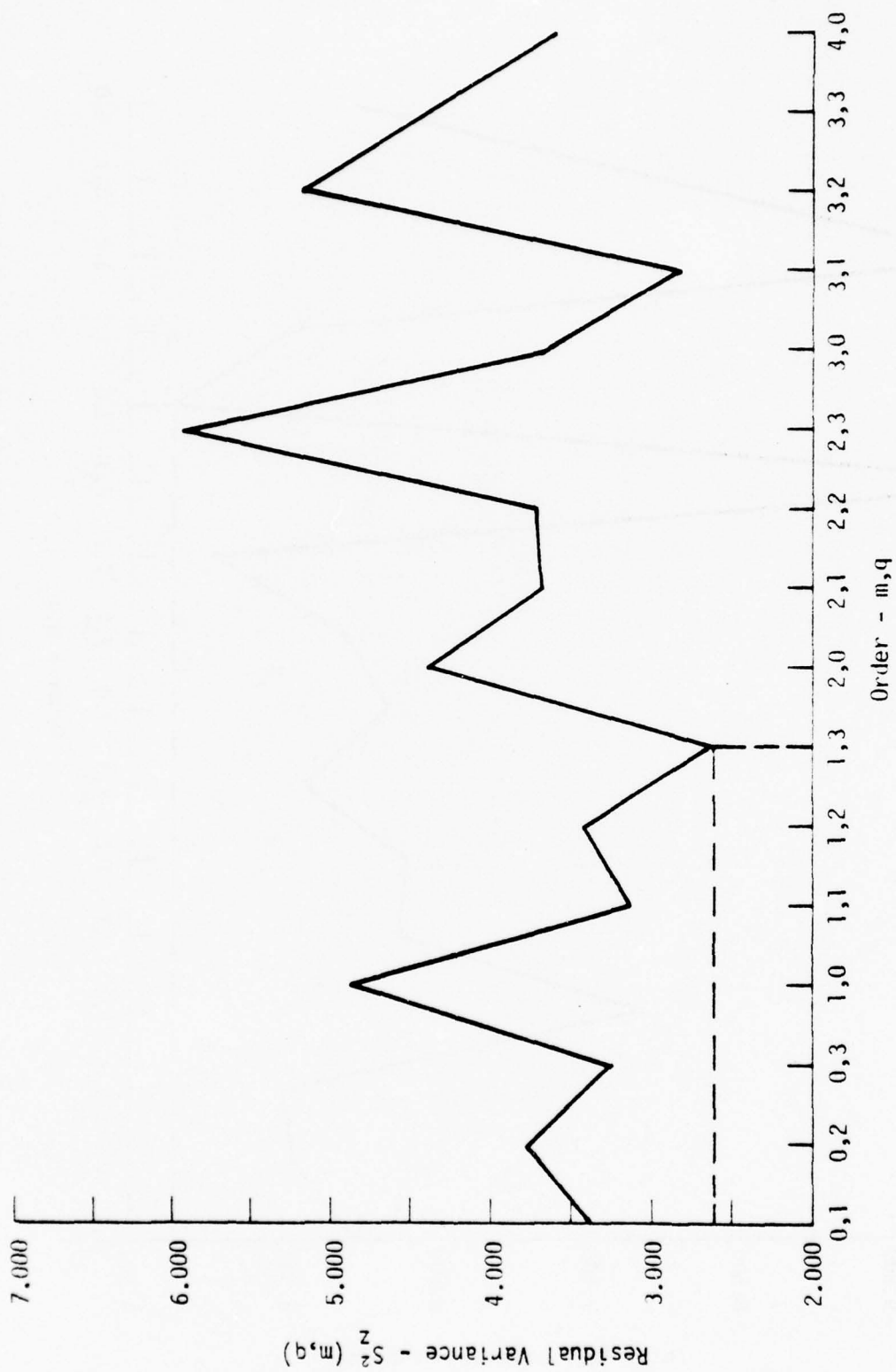
67

FIGURE 2.14 MODEL ORDER vs. RESIDUAL VARIANCE FOR THE MAN/MACHINE INTERFACE DATA, TELETYPEWRITER TERMINAL, 4,4 ENVIRONMENT
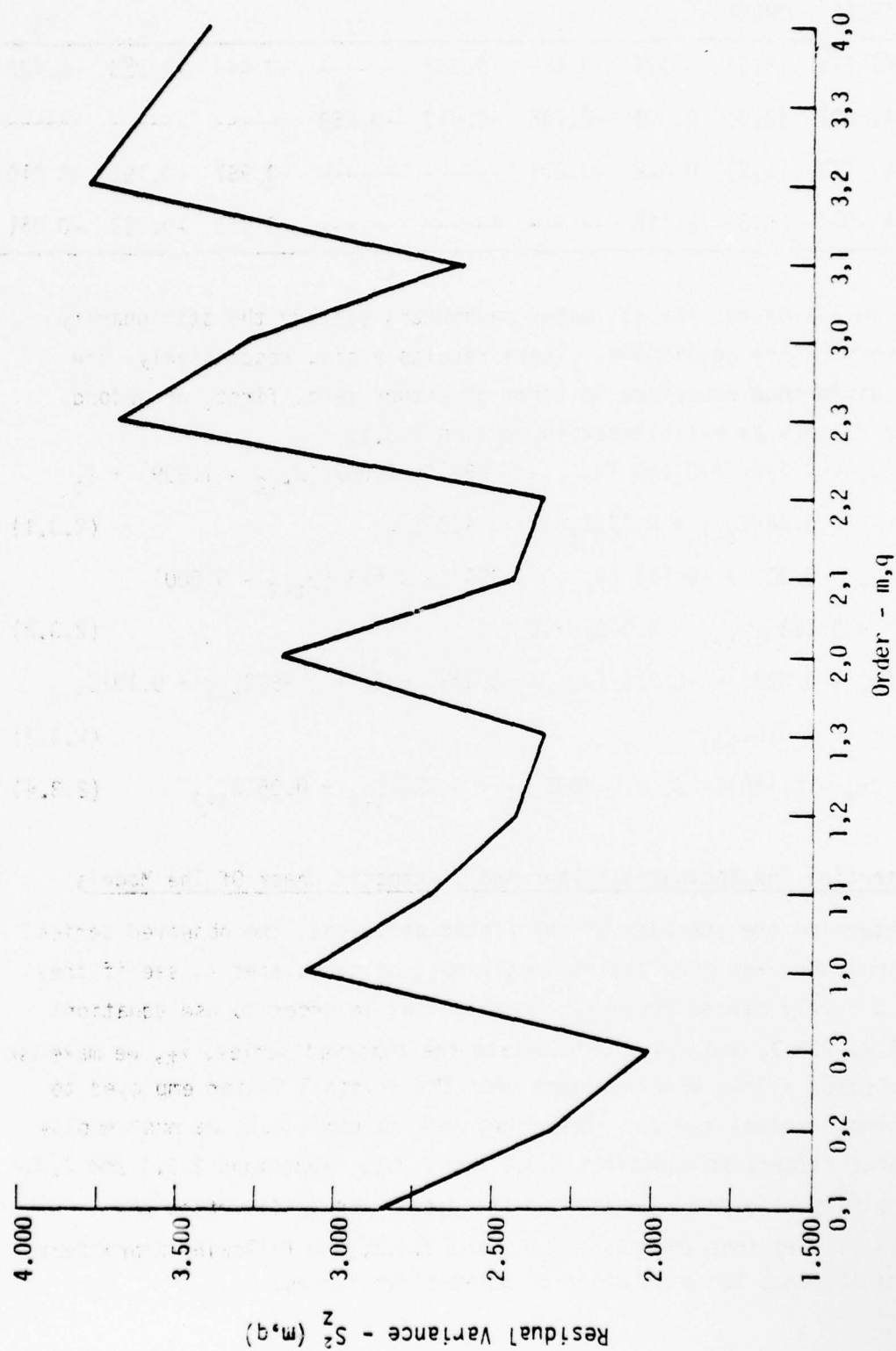
FIGURE 2.15 MODEL ORDER vs. RESIDUAL VARIANCE FOR THE MAN/MACHINE INTERFACE DATA, OPTICAL DISPLAY TERMINAL, 4,4 ENVIRONMENT

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Table 2.2 Approximate Least Squares Estimates
of the Best Model Parameters

MODEL / SERIES / ORDER	μ	α_1	α_2	α_3	β_1	β_2	β_3
i. {1,4} TTY (2,3)	1.699	0.660	0.367	------	-0.449	-0.223	-0.422
ii. {1,4} ODT (3,0)	0.000	-0.746	-0.613	-0.258	------	------	------
iii. {4,4} TTY (1,3)	0.028	-0.215	------	------	-0.957	-0.191	-0.016
iv. {4,4} ODT (0,3)	2.158	------	------	------	0.453	-0.023	-0.051

Note that in all cases, the estimated parameters satisfy the stationarity
and/or invertibility conditions. These results yield, respectively, the
following difference equations in terms of either zero, first, or second
difference filters as established in section 2.3.1:

i. $(x_t - 1.699) = 0.660 \, (x_{t-1} - 1.699) + 0.367 \, (x_{t-2} - 1.699) + Z_t$

$+ \, 0.449 Z_{t-1} + 0.223 Z_{t-2} + 0.422 Z_{t-3}$ (2.3.1)

ii. $(y_t - 0.000 = -0.746 \, (y_{t-1} - 0.000) - 0.613 \, (y_{t-2} - 0.000)$

$- \, 0.258 \, (y_{t-3} - 0.000) + Z_t$ (2.3.2)

iii. $(w_t - 0.028) = -0.215 \, (w_{t-1} - 0.028) + Z_t + 0.957 Z_{t-1} + 0.191 Z_{t-2}$

$+ \, 0.016 Z_{t-3}$ (2.3.3)

iv. $(x_t - 2.158) = Z_t - 0.453 Z_{t-1} + 0.023 Z_{t-2} + 0.051 Z_{t-3}$ (2.3.4)

2.3.3 Inserting The Backwards Filter And Diagnostic Check Of The Models

To determine the adequacy of the fitted processes, the observed series
must be simulated, and then the residuals must be calculated to see if they
behave as a purely random process. Recall, that in order to use equations
2.3.1, 2.3.2, 2.3.3, and 2.3.4 to simulate the observed series, x_t, we make use
of the *backwards filter* which depends upon the original filter employed to
transform the original series. Referring back to table 2.1, we must employ
the backwards filters to equations 2.3.2 and 2.3.3. Equations 2.3.1 and 2.3.4
required no filtering prior to fitting the data. Hence, inserting the
appropriate filters into equations 2.3.2 and 2.3.3, the following characteri-
zations are obtained for simulation of man-machine series:

i. $\hat{x}_t = -0.046 + 0.660x_{t-1} + 0.367x_{t-2} + Z_t + 0.449Z_{t-1} + 0.223Z_{t-2}$

$\quad\quad + 0.422Z_{t-3}$ (2.3.5)

ii. $\hat{x}_t = +0.254x_{t-1} + 0.133x_{t-2} + 0.355x_{t-3} + 0.258x_{t-4} + Z_t$ (2.3.6)

iii. $\hat{x}_t = 0.006 + 1.785x_{t-1} - 0.570x_{t-2} - 0.215x_{t-3} + Z_t + 0.957Z_{t-1}$

$\quad\quad + 0.191Z_{t-2} + 0.016Z_{t-3}$ (2.3.7)

iv. $\hat{x}_t = 2.158 + Z_t - 0.453Z_{t-1} + 0.023Z_{t-2} + 0.051Z_{t-3}$ (2.3.8)

Setting the unknown values of Z_t equal to their unconditional expectation of zero, we begin the simulation by initially assuming $x_1, \ldots, x_{m+d}$, are known; to simulate x_t, we assume $x_{t-1}, \ldots, x_{t-m-d}$, are known. Figures 2.16, 2.17, 2.18, and 2.19 show the simulated series along with the observed series. Clearly, we have a good fit of the estimated models with respect to the observed series.

To check if the residuals behave as a purely random process, the sample autocorrelation function of the residuals, $r_{zz}(k)$, was calculated for all lags. It is known that $r_{zz}(k) \sim N(0, {}^1/n)$. The calculated value of the standard deviation of $r_{zz}(k)$ was found to be 0.158, and the 95% confidence limits are $\pm (1.96)(0.158) = \pm 0.310$. At the 5% level of significance, one could expect $(.05)$ 40 or 2 out of the sample autocorrelations to lie outside the confidence interval for the (1,4) environment. The calculated confidence interval for the (4,4) environment is .347. Tables 2.3, 2.4, 2.5, and 2.6 show, for the four time series considered, that few of the sample autocorrelations lie outside the confidence limits.

Table 2.3 Sample Autocorrelation of the Residuals, $r_{zz}(k)$, for the Simulated (1,4) TTY Model. Confidence Interval = ± 0.310

Lags	Sample Autocorrelation of Residuals							
1-8	1.00	.30	.62	.51	.48	.55	.53	.52
9-16	.43	.31	.29	.32	.36	.32	.24	.17
17-24	.10	.13	.23	.28	.25	.19	.15	.19
25-32	.25	.26	.30	.27	.23	.20	.21	.26
33-40	.25	.23	.20	.17	.17	.14	.11	.06

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FILTERED MODEL:

$$(x_t - 1.699) = 0.660 (x_{t-1} - 1.699) + 0.367 (x_{t-2} - 1.699) + Z_t + 0.449 Z_{t-1} + 0.233 Z_{t-2} + 0.422 Z_{t-3}$$

ARMA FORECASTING MODEL:

$$\hat{x}_t = 0.660 x_{t-1} + 0.367 x_{t-2} + Z_t + 0.449 Z_{t-1} + 0.223 Z_{t-2} + 0.422 Z_{t-3} - 0.046$$

OBSERVED ——————

SIMULATED —*—*—*

Committed Errors

Four Second Time Intervals

FIGURE 2.16 SIMULATED MAN/MACHINE INTERFACE SERIES USING THE MIXED (ARMA) MODEL vs. THE OBSERVED INFORMATION FOR THE TELETYPEWRITER TERMINAL, 1,1 ENVIRONMENT

FILTERED MODEL:

$$y_t = -0.746\ y_{t-1} - 0.613\ y_{t-2} - 0.258\ y_{t-3} + Z_t$$

AUTOREGRESSIVE FORECASTING MODEL:

$$\hat{x}_t = 0.254\ x_{t-1} + 0.133\ x_{t-2} + 0.355\ x_{t-3} + 0.258\ x_{t-4} + Z_t$$

OBSERVED ——————

SIMULATED —*—*—*—

Committed Errors

Four Second Time Intervals

FIGURE 2.17   SIMULATED MAN/MACHINE INTERFACE SERIES USING THE AUTOREGRESSIVE MODEL vs. THE OBSERVED INFORMATION FOR THE OPTICAL DISPLAY TERMINAL, 1,4 ENVIRONMENT

FILTERED MODEL:

$$(w_t - 0.028) = - 0.215 \; (w_{t-1} - 0.028) + Z_t + 0.957 \; Z_{t-1} + 0.191 \; Z_{t-2} + 0.016 \; Z_{t-3}$$

ARMA FORECASTING MODEL:

$$\hat{x}_t = 1.785 \; x_{t-1} - 0.570 \; x_{t-2} - 0.215 \; x_{t-3} + Z_t + 0.957 \; Z_{t-1} + 0.191 \; Z_{t-2} + 0.016 \; Z_{t-3} + .008$$

OBSERVED ————

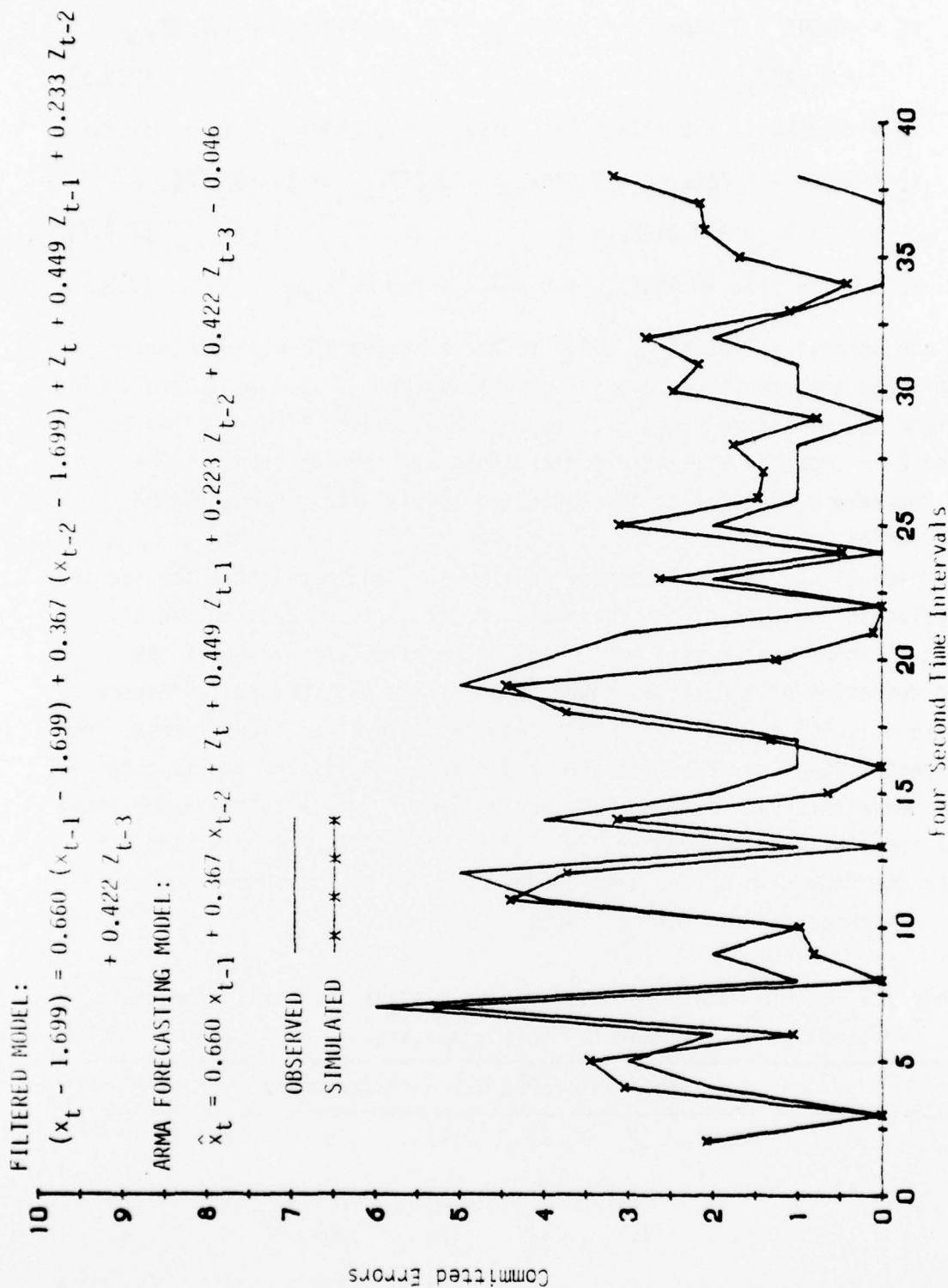SIMULATED —×——×——×—

Four Second Time Intervals

Committed Errors

FIGURE 2.18 SIMULATED MAN/MACHINE INTERFACE SERIES USING THE MIXED (ARMA) MODEL vs. THE OBSERVED INFORMATION FOR THE TELETYPEWRITER TERMINAL, 4,4 ENVIRONMENT

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FILTERED MODEL:

$$(x_t - 2.158) = Z_t - 0.453 \, Z_{t-1} + 0.023 \, Z_{t-2} + 0.051 \, Z_{t-3}$$

MOVING-AVERAGES FORECASTING MODEL:

$$\hat{x}_t = 2.158 + Z_t - 0.453 \, Z_{t-1} + 0.023 \, Z_{t-2} + 0.051 \, Z_{t-3}$$

OBSERVED ——————

SIMULATED —*—*—*—

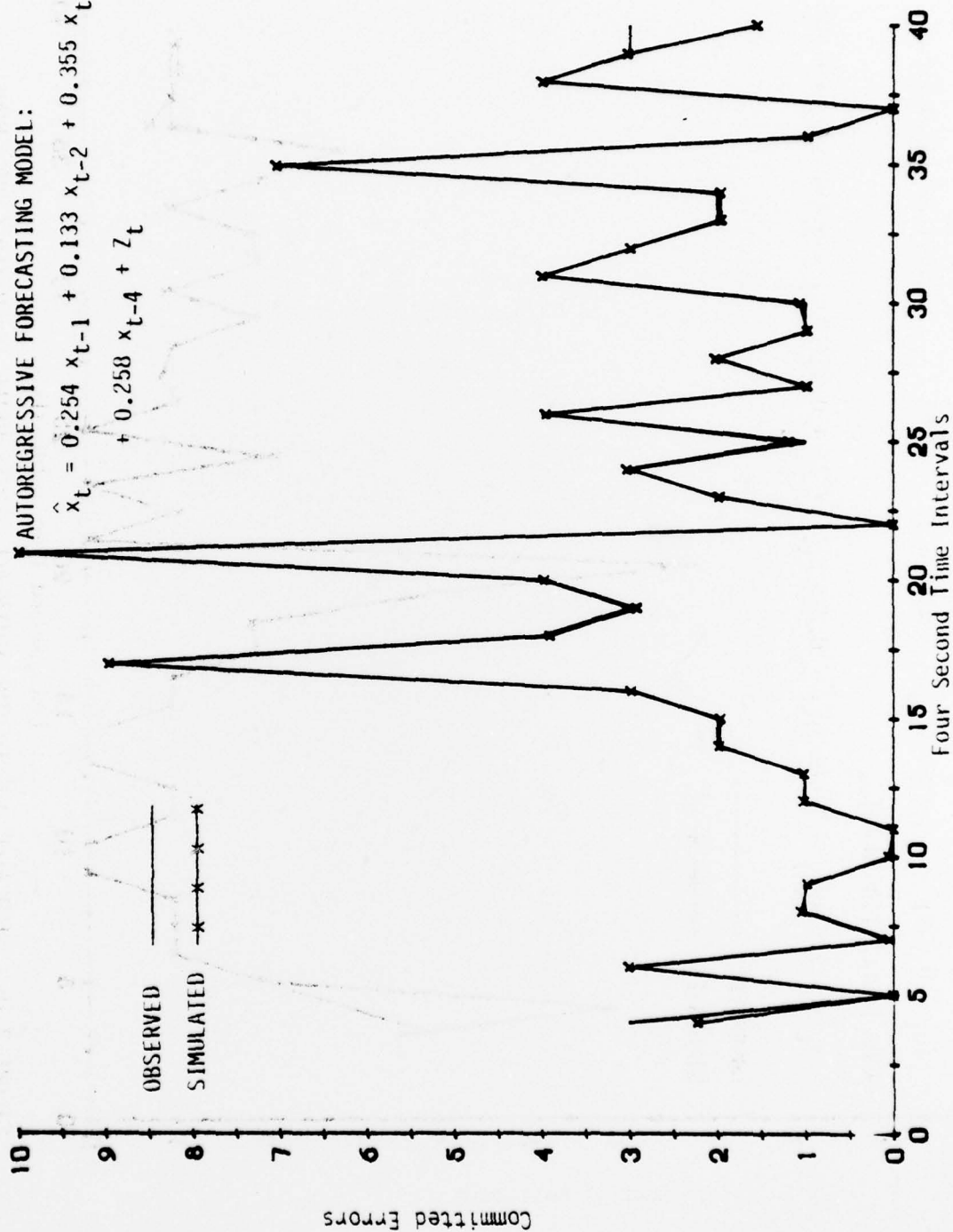Four Second Time Intervals

Committed Errors

FIGURE 2.19   SIMULATED MAN/MACHINE INTERFACE SERIES USING THE MOVING-AVERAGES MODEL vs. THE OBSERVED INFORMATION FOR THE OPTICAL DISPLAY TERMINAL, 4,4 ENVIRONMENT
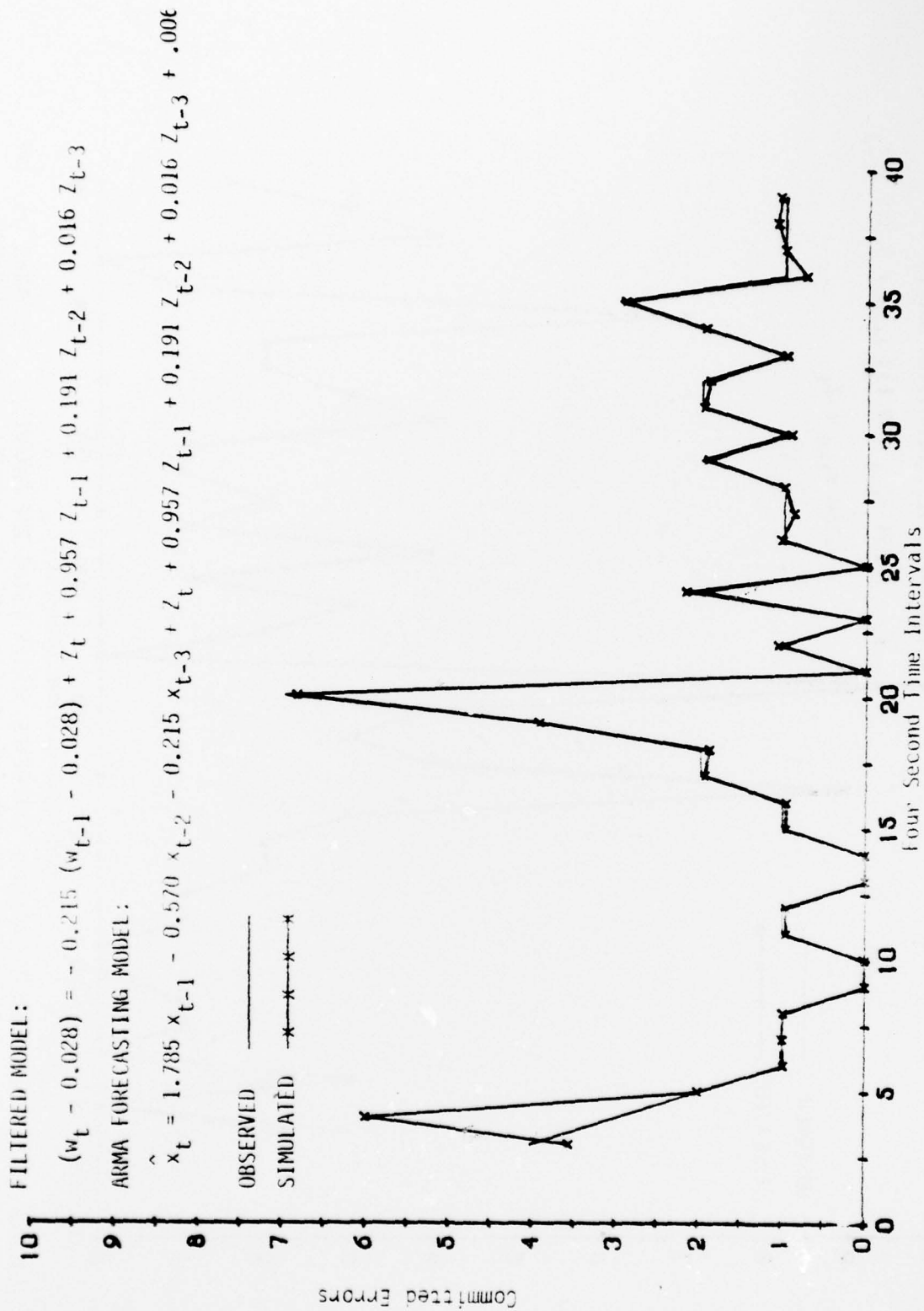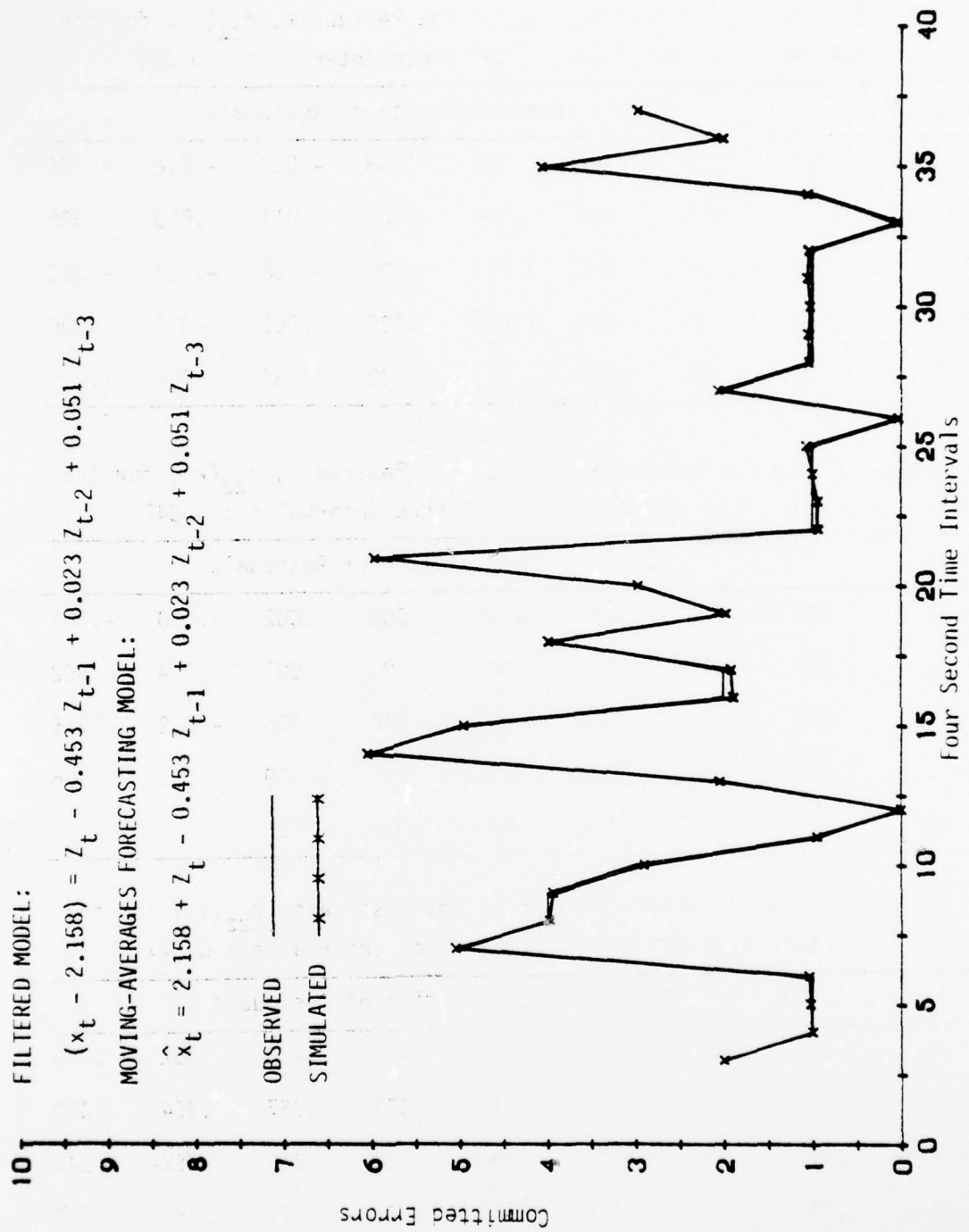

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Table 2.4 Sample Autocorrelation of the Residuals, $r_{zz}(k)$, for the
Simulated {1,4} ODT Model. Confidence Interval = ± 0.310

Lags	Sample Autocorrelation of Residuals							
1-8	1.000	.602	.362	.122	.008	-.012	-.010	-.006
9-16	-.001	-.008	.005	.000	.001	.011	.018	.005
17-24	.020	.023	.012	-.011	-.009	-.008	-.017	-.020
25-32	.009	.001	-.007	-.004	-.005	.003	.010	.005
33-40	.002	-.000	.013	.058	.129	.156	.218	.219

Table 2.5 Sample Autocorrelation of the Residuals, $r_{zz}(k)$, for the
Simulated {4,4} TTY Model. Confidence Interval = ± 0.347

Lags	Sample Autocorrelation of Residuals							
1-8	1.000	-.030	-.044	-.004	.002	-.002	-.000	-.002
9-16	-.007	-.000	-.002	.002	-.003	-.001	-.014	.002
17-24	.002	-.012	-.023	.041	-.006	.036	-.012	-.015
25-32	-.009	.001	.002	-.015	.008	-.009	.001	.006
33-38	-.010	-.047	.020	.060	.013	-.020		

Table 2.6 Sample Autocorrelation of the Residuals, $r_{zz}(k)$, for the
Simulated {4,4} ODT Model. Confidence Interval = ± 0.347

Lags	Sample Autocorrelation of Residuals							
1-8	1.000	.704	.692	.556	.477	.465	.452	.438
9-16	.424	.411	.399	.390	.379	.367	.354	.339
17-24	.327	.316	.307	.294	.282	.269	.258	.247
25-32	.237	.227	.214	.202	.189	.176	.163	.140
33-38	.097	.069	.090	.066	.029	.047		

Hence, we conclude that the residuals for the four man-machine interface characterizations do behave as a purely random process, and thus, equations 2.3.5, 2.3.6, 2.3.7, and 2.3.8 give satisfactory representation of the observed series.

2.4   FORECASTING AND UPDATING

To forecast "ℓ" time slots in advance of any origin, $t > m + d$, we replace t with $t + \ell$ in equations 2.3.5, 2.3.6, 2.3.7, and 2.3.8. Having made the appropriate substitution, we obtained the following minimum mean square error forecasting models:

  i. for the $(1,4)$ TTY characterization:

$$\hat{x}_t(\ell) = -0.046 + 0.660x_{t+\ell-1} + 0.367x_{t+\ell-2} - 0.449Z_{t+\ell-1}$$
$$+ 0.223Z_{t+\ell-2} + 0.422Z_{t+\ell-3} \; , \tag{2.4.1}$$

  ii. for the $(1,4)$ ODT characterization:

$$\hat{x}_t(\ell) = + 0.254x_{t+\ell-1} + 0.133x_{t+\ell-2} - 0.355x_{t+\ell-3} + 0.258x_{t+\ell-4} \; , \tag{2.4.2}$$

  iii. for the $(4,4)$ TTY characterization:

$$\hat{x}_t(\ell) = 0.006 + 1.785x_{t+\ell-1} - 0.570x_{t+\ell-2} - 0.215x_{t+\ell-3}$$
$$+ 0.950Z_{t+\ell-1} + 0.191Z_{t+\ell-2} + 0.016Z_{t+\ell-3} \; , \tag{2.4.3}$$

  iv. for the $(4,4)$ ODT characterization:

$$\hat{x}_t(\ell) = 2.158 - 0.453Z_{t+\ell-1} + 0.023Z_{t+\ell-2} + 0.051Z_{t+\ell-3} \tag{2.4.4}$$

In brief, suppose that at $t = 20$, a forecast $\hat{x}_{22}$ is made for the $t = 22$ time slot. x_{22} may be updated to become the $t = 21$ origin forecast of x_{22} by adding a constant multiple of the one-step ahead forecast error, $\theta_\ell Z_{t+1} = \theta_1 Z_{21}$, to the $t = 20$ origin forecast of x_{22}. That is: $x_{22}^* = \hat{x}_{22} + \theta_1 Z_{21}$. The forecast error for this case would be: $Z_{21} = x_{21} - \hat{x}_{21}$ and $\theta_\ell = \theta_1$ is given by equations (2.2.19) and (2.2.20). The updating is done when $x_{t+1} = x_{21}$ becomes available. Tables 2.7, 2.8, 2.9, and 2.10 show the ℓ steps ahead forecasts (up to $\ell = 11$) at an arbitrary $t = 20$ origin, with updating, along with their 95% confidence intervals.

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Table 2.7 Forecasted Values of the {1,4} TTY Series at Origin t = 20, and Updating Under the Assumption x_{21} Becomes Available

TIME INTERVAL	ACTUAL ERRORS	LEAD TIME	FORECAST	95% PROBA-BILITY LIMITS	UPDATED FORECAST
20	3.0	--	---	----	----
21	5.0	1	4.4	± .289	----
22	4.0	2	1.2	± .363	1.35
23	3.0	3	.1	± .484	.22
24	.0	4	.0	± .637	.00
25	2.0	5	2.6	± .809	2.31
26	.0	6	.5	± .907	.31
27	2.0	7	3.1	± .997	2.89
28	1.0	8	1.5	±1.084	1.25
29	1.0	9	1.4	±1.166	1.17
30	1.0	10	1.8	±1.247	1.58
31	.0	11	.8	±1.326	.57

Table 2.8 Forecasted Values of the {1,4} TTY Series at Origin T = 20, and Updating Under the Assumption x_{21} Becomes Available

TIME INTERVAL	ACTUAL ERRORS	LEAD TIME	FORECAST	95% PROBA-BILITY LIMITS	UPDATED FORECAST
20	3.0	--	---	----	----
21	4.0	1	3.9	± .405	----
22	10.0	2	10.1	± .412	9.89
23	.0	3	.0	± .447	.00
24	2.0	4	1.9	± .486	2.07
25	3.0	5	3.0	± .501	3.16
26	1.0	6	1.2	± .520	1.03
27	4.0	7	3.9	± .545	4.07
28	1.0	8	1.0	± .567	1.00
29	2.0	9	2.0	± .584	2.17
30	1.0	10	1.0	± .603	.89
31	1.0	11	1.0	± .620	.90

Table 2.9 Forecasted Values of the {4,4} TTY Series at Origin t = 20,
and Updating Under the Assumption x_{21} Becomes Available

TIME INTERVAL	ACTUAL ERRORS	LEAD TIME	FORECAST	95% PROBA-BILITY LIMITS	UPDATED FORECAST
20	4.0	--	---	----	----
21	7.0	1	6.8	± .297	----
22	.0	2	.0	± .752	.090
23	1.0	3	1.1	±1.385	1.005
24	.0	4	.6	±2.242	.010
25	2.0	5	2.2	±3.236	2.100
26	.0	6	.0	----	.000
27	1.0	7	1.0	----	1.000
28	1.0	8	.9	----	.960
29	1.0	9	1.0	----	1.000
30	2.0	10	1.9	----	2.100
31	1.0	11	1.4	----	.900

Table 2.10 Forecasted Values of the {4,4} ODT Series at Origin t = 20,
and Updating Under the Assumption x_{21} Becomes Available

TIME INTERVAL	ACTUAL ERRORS	LEAD TIME	FORECAST	95% PROBABILITY LIMITS	UPDATED FORECAST
20	2.0	--	---	----	----
21	3.0	1	2.96	±.275	----
22	6.0	2	5.96	±.278	6.010
23	1.0	3	.94	±.233	1.081
24	1.0	4	.94	±.307	1.174
25	1.0	5	.99	±.307	.976
26	1.0	6	1.07	±.307	1.025
27	.0	7	.03	±.307	.000
28	2.0	3	2.07	±.307	2.003
29	1.0	9	1.05	±.307	.978
30	1.0	10	1.04	±.307	.959
31	1.0	11	1.03	±.307	.940

Clearly, the forecasts for $\ell < 11$ are quite good for the AR, MA, and ARMA realizations. For the ARMA process ((1,4) TTY series, table 2.7), the forecasts for $\ell = 1, 2, 3$, are not very good. As ℓ is increased, the variance and the confidence limits increase correspondingly. Also, in general, one can say that as ℓ increases beyond $m + d$ (m is the order of the AR component and d is the order of the filter), the forecasts begin to over-estimate the future values. This fact and the results of table 2.7 are strongly indicative of the need for updating as new information become available. As seen in the tables above, accuracy increases with updating.

2.5 SUMMARY AND CONCLUSIONS

The procedure developed in section 2.2 and illustrated in detail in sections 2.3 and 2.4, was followed precisely in characterizing the man-machine interface data. The information acquired through an extensive environmental experiment involved two different communications terminals and six experienced communications-center operators. The operators were tested under sixteen different combinations of ambient light and acoustic noise (refer to section 1.2). Because of the extensive data base, the two most critical environmental combinations, namely, (1,4) and (4,4), were addressed in this treatise. Clearly, the time-series characterization of the data is very promising from the point of view of affording to the communications system designer and planner a means to predict the human element of the total communications system architecture. It has been shown that the realizations obtained were very adequate in characterizing the underlying process of error performance.

In review,

a. for the (1,4) environment, *teletypewriter terminal*, we obtained the (2,3) ARMA process:

$$\hat{x}_t = -0.046 + 0.660x_{t-1} + 0.367x_{t-2} + Z_t + 0.449Z_{t-1} + 0.223Z_{t-2} + 0.422Z_{t-3} ;$$

b. for the (1,4) environment, *optical display terminal*, the (3,0) AR process obtained was:

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$$\hat{x}_t = 0.254x_{t-1} + 0.133x_{t-2} + 0.355x_{t-3} + 0.258x_{t-4} + Z_t \quad ;$$

c. for the $\{4,4\}$ environment, *teletypewriter terminal*, we obtained another ARMA process of order $(1,3)$:

$$\hat{x}_t = 0.006 + 1.785x_{t-1} - 0.570x_{t-2} - 0.215x_{t-3} + Z_t + 0.950Z_{t-1}$$
$$+ 0.191Z_{t-2} + 0.016Z_{t-3} \quad ;$$

d. and, finally, for the $\{4,4\}$ environment, *optical display terminal*, the $(0,3)$ MA process obtained was:

$$\hat{x}_t = 2.158 + Z_t - 0.453Z_{t-1} + 0.023Z_{t-3} + 0.051Z_{t-3} \quad .$$

One of the implied features of this chapter is that for each environmental combination, no *common* realization, either ARMA, MA, or AR, was obtained to characterize operator performance. One can conclude, therefore, that even with an adequately developed procedure for analysis, more than one characterization is required to evaluate the human subsystem in sophisticated communications systems. The procedures developed in section 2.2 clearly provide a realistic view of the complex man-machine interface that occurs in current communications systems.

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